Inventory model is a mathematical representation of inventory system which is used for determining the optimal operating policies for inventory management and control. Inventory models are extensively used for planning and scheduling several inventory systems arising at places like production processes, market yards, assembly lines, warehouses, etc,. The inventory models are broadly classified into two categories based on the nature of commodity. They are (i) inventory models for deteriorating items and (ii) inventory models with infinite lifetime. The inventory models for deteriorating items are again divided into two groups namely (i) inventory models for deteriorating items with fixed lifetime and (ii) inventory models for deteriorating items with random lifetime. Inventory models provide the basic frame work for analyzing several production systems. The inventory models are further categorized into two groups namely, (i) economic order quantity models (EOQ models) and (ii) economic production models (EPQ models).

The EPQ models are more common in production and manufacturing processes, warehouses, etc. In EPQ models the major string is relaxing some of the assumptions regarding production (replenishment), nature of the commodity and demand pattern (Osteryoung, et al**.** (1986)). Recently much emphasis was given for analyzing EPQ models for perishable items. Deterioration is a natural phenomenon of several commodities. The deterioration is highly influenced by several random factors like storage facility, temperature, environmental conditions, quality of raw material etc. Several authors various EPQ models for deteriorating items with various assumption on lifetime of commodity.

Nahimias (1982), Raafat (1991), Goyal and Giri (2001), Ruxian Lie, et al. (2010) and Pentico and Drake (2011) have reviewed the literature on inventory model for deteriorating items. To develop the EPQ models it is needed to ascribe a probability distribution to the lifetime of commodity. Ghare and Schrader (1963), Shah and Jaiswal (1977), Cohen (1977), Aggarwal (1978), Dave and Shah (1982), Pal (1990), Kalpakam and Sapna (1996), Giri and Chaudhuri (1999) assumed that the lifetime of commodity follows an exponential distribution. Tadikamalla (1978) assumed gamma distribution to the lifetime of commodity, Covert and Philip (1973), Philip (1974), Goel and Aggarwal (1980), Venkata Subbaiah, et al. (2004) assumed Weibull distribution to the lifetime of commodity. Nirupama Devi, et al. (2001) developed inventory models with mixture of Weibull distribution for the lifetime of commodity, Srinivasa Rao, et al. (2005) developed inventory model with generalized Pareto lifetime, Xu and Li (2006) developed a two-warehouse inventory model for deteriorating items with time-dependent demand, Rong, et al. (2008) studied a two-warehouse inventory model for deteriorating items with partially/fully backlogged shortages and fuzzy lead time, Srinivasa Rao, et al. (2009) studied an inventory model for deteriorating items having additive exponential life time and selling price dependent demand rate, Chang and Lin (2010) studied an inventory model for deteriorating items with stock dependent demand, Biswajit Sarkar (2012) developed an EOQ model with delay in payments and stock dependent demand in presence of imperfect production. However, in all these EOQ models, it is assumed that the replenishment is instantaneous with finite rate.

Mukarjee and Pal (1986), Sujit and Goswami (2001), Goyal and Giri (2003) developed inventory models with finite rate of replenishment (production). Panda and Chatarjee (1987), Mandal and Phajudar (1989) and Sana, et al. (2004) developed inventory models with uniform rate of replenishment (production). Perumal and Arivarignan (2002) considered two rates of production in an inventory model. Pal and

Mandal (1997) and Sen Chakrabarty (2007) developed alternating replenishment rates. Lin, et al. (2006), Maiti, et al. (2007), Hu and Liu (2010), Uma Maheswara Rao, et al. (2010) have developed inventory models for deteriorating items with constant rate of production (replenishment). Venkata Subbaiah, et al. (2011) have developed EPQ model with alternating rate of replenishment, Essay, et al. (2012) have developed EPQ models with stock dependent production and Weibull decay.

However, in many production lot size models the replenishment rate is not constant or uniform and will have a variable rate of production (replenishment), since the production or replenishment is influenced by several random factors like transportation, quality of raw materials, availability, packaging, environmental conditions etc. For example in case of seafood's and agricultural products the uncertainty in the yield effects the replenishment. Also it can be observed several production processes dealing with perishable items will have a variable rate of replenishment. For modeling this sort of situation it is needed to consider the replenishment (production) is random and follows a probability distribution. Very little work has been reported in literature regarding economic production quantity models with random production except the works of Sridevi, et al. (2010) and Srinivasa Rao, et al. (2010), who have develop and analyzed inventory models for deteriorating items with random replenishment. They assumed that the deterioration rate is constant.

Since several of the items are having variable rate of decay and the decay starts only after certain period of time it is reasonable to consider that lifetime of commodity is random and follows Pareto distribution. The Pareto distribution includes uniform distributions as limiting cases. With this motivation in this thesis we develop and analyze some EPQ models for deteriorating items with Exponential production and Pareto decay having various patterns of demand.

First we develop and analyze an inventory model for deteriorating items with the assumption that the replenishment (production) is random and follows a Exponential distribution. It is further assumed that the lifetime of the commodity is random and follows a Pareto distribution. It is also assumed that the demand rate is a function of selling price and is of the form $f(s) = a - bs, \quad 0 < b < 1$, where ‘a’, ‘b’ are constants, ‘s’ is the selling price. Assuming shortages are allowed and fully backlogged the inventory model is derived. This model is extended to the case of without shortages.

Another variation in this model is also investigated by considering that the demand is a function of time. It is a common belief that time has a tremendous influence on the demand of the commodity. Here, it is assumed that the demand follows power pattern with indexing parameter. For different values of indexing parameter it includes different types of demand. Assuming the shortages are allowed and fully backlogged, the inventory model with Exponential rate of production and generalized Pareto decay having time dependent demand is analyzed. This model is extended to the case of without shortages.

Both selling price and time put together influence the demand in some commodities like edible oils and agricultural products. Hence another economic production quantity model with Exponential rate of production and Pareto decay having both selling and time dependent demand is developed and analyzed for

with and without shortages.

In all these models, using the differential equations the instantaneous state of inventory at time‘t’, the stock loss due deterioration, production (order) quantity are derived. With suitable cost considerations, the total cost function per unit time and the profit rate function are derived. By optimizing the total cost function or profit rate function the optimal production schedules and ordering quantities are derived. The sensitivity of the model with respect to parameters and costs are also analyzed. Through numerical studies, the solution procedures for optimal policies are demonstrated. These models also include some of the earlier models as particular cases for specific or limiting values of the parameters. The models derived in this thesis are having practical utilization in seafood’s industries, food and vegetable markets, cement, chemical industries warehouses and market yards etc., where the commodity lifetime is random and follows Pareto distribution and the production is governed by distribution.

In this project economic production quantity models are developed and analyzed for a single commodity under consideration. It is possible to develop EPQ models for multiple commodities using random production (variable rate of production). Throughout the project it is assumed that the money value remain constant over the period of time i.e. the inflation has no influence on the models. It is also possible to develop and analyze the EPQ models developed in this thesis with inflation (time values of money) which require further investigation.

In this project the production level inventory models for deteriorating items are developed with the assumption that the production is governed by laws of chance and the variable production can be characterized with Exponential distribution. It is also considered that the lifetime of commodity is random and follows Pareto distribution. Different demand patterns are considered to provide spectra of EPQ models. These EPQ models are having practical utilization in scheduling the production processes dealing with perishable commodities like sea food’s jelly jams, cement, edible oil, agricultural products, chemicals and pharmaceuticals etc.

The lifetime distribution of the models also includes the distributions such as uniform as limiting cases. The Exponential rate of production can include increasing, decreasing and variable rates of production which provide a flexibility to implement the model for different situations. The production/operational managers can estimate the production parameters and deteriorating parameters from the historical date available in the records. The estimation of the cost can also be inferred from records of marketing and stores. These models include several of the earlier models as particular cases for limiting or specific values of the parameters.

It is highly possible to develop many more economic production quantity models with plausible conditions in order to utilize the resources more optimally, efficiently and effectively.

Technical Report submitted to UGC Minor Research Project

SOME ECONOMIC PRODUCTION QUANTITY MODELS WITH EXPONENTIAL RATE OF PRODUCTION AND PARETO DECAY

UGC Minor Research Project sanction **F.No.MRP-7082/16 (UGC-SERO)-Dated 05-03-2018**

Link No: 7082

(Technical Report No: AITAM/BS&H/UGC/Minor Res. Proj/2018/March/05)

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CERTIFICATE

This is to certify that the project work titled ***"SOME ECONOMIC PRODUCTION QUANTITY MODELS WITH EXPONENTIAL RATE OF PRODUCTION AND PARETO DECAY"*** is carried out by me and was not submitted for partial/full financial assistance to any other funding agency.

A LAKSHMANA RAO
Principal Investigator

ACKNOWLEDGEMENTS

It is indeed with a great sense of pleasure and immense sense of gratitude that I acknowledge the help of these individuals. I am highly indebted to the **University Grants Commission – SERO, Hyderabad** for considering my proposal and providing financial assistance.

I am highly indebted to **Prof. V .V. Nageswara Rao**, Director of Aditya Institute of Technology And Management(A) and **Dr. A. Srinivasa Rao**, Principal for the facilities provided to accomplish this project. I am obliged to **Dr K.B Madu Sahu**, Director R&D and **Dr R Santhi Kumar** and **Sri S Bhaskara Babu** team of R&D for their encouragement.

I would like to thank Dr**. G Vasanthi**, Head of the Department, Dept. of BS&H for her constructive support throughout the project.

My heartful thanks are due to all my colleagues in the department for their keen interest and great support throughout the project.

Last, but not least, I am beholden to my parents Late **Sri Suryaprakasa Rao** and **Smt. Ratnalamma** for the love and affection bestowed on me. I remain thankful to my wife **Smt. Rajeswari** and daughter **Chy. Chow. Harsha Vardhini** and son **Chy. Haricharan** and all other affectionate members of our family who extended all cooperation for enabling me to pursue my research work.

A.LAKSHMANA RAO
Principal Investigator

CONTENTS

CHAPTER – I

INTRODUCTION

1. INTRODUCTION

1.1 INVENTORY MODELS FOR DETERIORATING ITEMS:

Inventory model is a mathematical representation of inventory system which is used for determining the optimal operating policies for inventory management and control. Inventory models are extensively used for planning and scheduling several inventory systems arising at places like production processes, market yards, assembly lines, warehouses, etc,. The inventory models are broadly classified into two categories based on the nature of commodity. They are (i) inventory models for deteriorating items and (ii) inventory models with infinite lifetime. The inventory models for deteriorating items are again divided into two groups namely (i) inventory models for deteriorating items with fixed lifetime and (ii) inventory models for deteriorating items with random lifetime.

Starting from the first inventory models for deteriorating items by Wihitin (1957) much work has been reported regarding inventory problems dealing with deteriorating items. Raafat (1991), Goel and Giri (2001), Ruxien Li, et al. (2010), Pentico and Drake (2011) have reviewed the inventory models for deteriorating items. Deterioration is usually defined as the damage, decay, spoilage, evaporation and obsolescence of item. In real life many items deteriorate due to inherent nature, for example fruits, vegetables, food items, seafood's, agricultural products, textiles, chemicals, medicines, electronic components, cement, fertilizers, oils, gas etc., are some of the deteriorating items which are kept in inventory at various places.

The conventional inventory models are developed to formulate economic order quantity assuming the replenishment is instantaneous. These economic order quantity models are much useful for scheduling the inventory systems at fruit and vegetable markets, stock yards, super markets etc. However, in production processes, manufacturing units, ware houses etc., the replenishment is not instantaneous and it is having a finite rate.

To analyze this type of situations economic production quantity models are developed. In economic production quantity model, the optimal operating policies are concerned with the determination of quantity to be produced (to be ordered) time to start production (production uptime) and time to stop production (production downtime). Several EPQ models have been developed and analyzed with various assumptions on constituent processes of the inventory systems namely, demand, deterioration and production. Among these factors production is considered to be very important. Due to various random factors influencing the production, the production is to be considered as random.

Very little work has been reported in literature regarding EPQ models with random production even though they are much useful for analyzing many inventory systems which have variable rate of production (replenishment). Hence, in this thesis we fill the gap in this area of research by developing and analyzing some inventory models for deteriorating items with random production and generalized Pareto decay having different demand patterns.

1.2 REVIEW ON EOQ INVENTORY MODELS FOR DETERIORATING ITEMS WITH RANDOM LIFE TIME:

In this section, we briefly review some of the important contributions in inventory models for deteriorating items. Decay or deterioration of physical goods is a common phenomenon in many inventory systems. Market yards, warehouses, production processes, transportation, cargo handling, food processing units, fruits, vegetable markets, cement, petro chemicals and film industries are few places which deal the inventory of such items. Deteriorating inventory models have been widely studied in recent years. The analysis of inventory problems for deteriorating items began with **Ghare and Schrader** (1963) who developed an economic order quantity model with constant rate of decay. They have derived an equation for the optimum order quantity as a function of the inventory cycle time. They pointed the importance of considering the effect of decay in inventory analysis by noting the potentials for cost saving measures and the improvements in inventory reordering policy.

Brown, et al. (1964) studied an inventory model to study the problem under stochastic obsolescence in supply system with particular reference to spare parts of naval aircraft. The response to obsolescence by means of Bayesian procedure and the optimal inventory policies using dynamic programming technique are derived. **Pierskalla and Roach** (1972) obtained optimal issuing policies for some particular classes of perishable inventory under several possible objective functions. The items in stock are grouped into four categories according to shelf and proved that for most of the objective functions; the optimal policy is to issue the oldest unit, which will satisfy the demand.

Covert and Philip (1973) studied an economic order quantity model for variable rate of deterioration by assuming a two- parameter Weibull distribution. **Philip** (1974) extended the model by considering variable deterioration rate with three parameter Weibull distribution. **Fries** (1975) further extended the classical single- item, multi-period inventory model to the case, where an item in storage perishes exactly at fixed number of periods after its receipt on order. The optimal ordering quality and the properties of the solution for any fixed number of periods are derived.

Nahimias (1975a, 1975b) considered the inventory models for describing optimal ordering policies for perishable inventory as multidimensional dynamic program. He derived the formulas for the quantity of the order which will outdate for the case where demand has stationary Erlang distribution. He modified the one-period model to yield reasonable approximation to the stationary optimal policy. By using dynamic programming he extended the model to the multi-period case. He analyzed the structure of the optimal ordering policies by considering the cost that has to be incurred at the time of out dating.

Cohen (1977) studied the problem of joint pricing and ordering policy for an exponentially decaying inventory with known demand. **Shah and Jaiswal** (1977) developed an order-level inventory model for deteriorating items with a constant rate of deterioration under deterministic and probabilistic demands and instantaneous delivery. **Pandu** (1978) considered an inventory model in which inventory is depleted by demand and deterioration where the time to deterioration follows gamma distribution. The EOQ formula was derived under conditions of constant demand, instantaneous delivery and no shortages are permitted. **Goel and Aggarwal** (1980) developed an algorithm for determining the optimal pricing and ordering policy for three parameter Weibull rate of determining inventory assuming the replenishment is instantaneous and demand is a function of selling price.

Dave and Patel (1981) considered an inventory model in which the demand rate is changing linearly with time and no backlogging is allowed. They assumed the deterioration rate to be constant with finite planning horizon and equal replenishment cycles. They obtained expressions for determining the optimum number of replenishments. **Deuermeyer** (1979) studied a model for obtaining the properties of optimal production policy for system composed of two production process having independent random demands and different life times. **Deuermeyer** (1980) developed a single period model for a class to multi- product perishable inventory systems, where demands are independent and lifetime of product is fixed. It is shown that the optimal policy for this substitution property and the rate of substitution is age- dependent.

Aggrawal and Goel (1982) developed an order level inventory model for deteriorating items having power pattern demand where constant fraction of on hand inventory deteriorates per each unit of time. Both deterministic as well as probabilistic cases of demand with and without shortages were considered. **Nahimias** (1982) reviewed relevant literature on the problem of determining suitable ordering policies for both fixed life perishable inventory and inventory subject to continuous exponential decay. He considered both deterministic and stochastic demand for single and multiple products. Both optimal and suboptimal order policies were discussed. The review of application of the models to blood bank management was also included.

Kaspi and Perry (1983) considered the blood bank inventory system, in which both arrival of items and demand are stochastic and items stored have finite lifetimes. **Kaspi and Perry** (1984) studied an inventory system for perishable items with renewal input and Poisson output. Three models with the assumptions that the stored items have finite and fixed life times are discussed. **Sachan** (1984) considered an economic order quantity model, in which items of inventory deteriorate at a constant rate and shortages are allowed for the situation of fixed cycle time and increasing (or decreasing) level of order quantities for the time proportional demand.

Pandit and Rao (1984) considered a number of demands for food items during a cycle of an inventory system with initial stock Q and items in stock deteriorating stochastically over time. It is assumed that the demands occur as a Poisson process and the lot is replenished with zero lead-time, making cycle time itself a random variable. **Bhattacharjee** (1985) considered the inventory problems of perishable commodity and uncertainty in the amount received against order under conditions of relative scarcity.

Goyal (1985) obtained models for obtaining the economic order quantity (EOQ) for an item for which the supplier permits a fixed delay in setting the amount owed to him. An example was given to illustrate the method. **Backer and Urban** (1988) analyzed continuous, deterministic cases of an inventory system with instantaneous replenishment over an infinite planning horizon in which the demand rate of an item was inventory level-dependent. **Perry** (1985) studied an inventory system for perishable commodities, in which the life times of the items are i.i.d (identical independently distributed) random variable with finite mean.

Panda and Chatterjee (1987) developed an inventory model; continuous in units and discrete in time, for deteriorating items, the model under know uniform demand and uniform replenishment is discussed. **Datta and Pal** (1988) considered an EOQ model with a power pattern demand and variable rate of deterioration assumed to be a special form of two parameter Weibull distributions. Shortages were allowed and replenishment rate was assumed to be infinite.

Kalpakam and Arivarignan (1988) considered a continuous review (s, S) inventory system, where items are removed from stock one at time either due to random demand or due to random failure of item. It is assumed that the demand for an item occurs in Poisson manner and lifetime of any item is negative exponential. **Pal** (1989) developed an inventory model for items with a constant rate of decay, when lead-time is random and having an exponential distribution. The policy here is to place an order with a supply centre whenever a demand exists or an item in stock deteriorates.

Upendra (1990) considered a probabilistic scheduling period inventory system for continuously deteriorating items, where demand occurs instantaneously at the beginning of scheduling period. Here, shortages are not allowed and deterioration is assumed as constant fraction of on-hand inventory in a lot-size system that allows partially backlogging and partially lost selling. **Datta and Pal** (1990) discussed an infinite time horizon deterministic inventory model without shortages, where the demand rate at any instant depends on the on-hand inventory (stock level) at that instant down to a certain stock level and then it becomes constant for the remaining period of the cycle.

Goswami and Chaudhuri (1991) studied the inventory replenishment problem for a deteriorating item with linearly time – varying demand, finite shortages cost and equal replenishment intervals. They developed a solution procedure to determine the number of reorders, the interval between successive reorder points and the shortage interval in an optimal manner as to minimize the total system cost. **Raafat** (1991) presented a review of inventory literature for the deteriorating (decaying) inventory models.

Urban (1992) studied a deterministic inventory system over an infinite time horizon with demand rate as a function of inventory- level followed by a constant demand rate, in which the terminal condition of zero inventories at the end of the order cycle is not imposed on the system. Replenishment was assumed instantaneous with a known, constant lead time.

Ishii (1993) discussed an inventory model for a single perishable product with two types of customers and different selling prices. **Fujiwara and Perera** (1993) presented on model for perishable products, which consider continuous deterioration of the utility of a product and introduced an exponential penalty constant function as a measurement of utility deterioration. **Pakkala and Achary** (1994) considered an order-level inventory model for deteriorating items, when two separate warehouses are used. When stock level excess the capacity of own warehouse (OW), an additional rented warehouse (RW) is used. Here, the demand is uniform, replenishment rate is finite and shortages are permitted.

Gore and Shah (1994) developed a deterministic order level inventory system for items having exponential decay with shortages and uncertainty in quantity received. Formula for total expected cost and ordering quality are derived. **Padmanabhan and Vrat** (1995) presented EOQ models for perishable items under stock dependent selling rate, instantaneous replenishment and constant rate of deterioration. **Wee** (1995) considered a replenishment policy for deteriorating product where demand declines exponential over a fixed time horizon. Deteriorating rate was assumed to be constant and complete backordering of demand was allowed. An extension of the model for partial back ordering was also illustrated.

Abad (1996) formulated a model of dynamic pricing and lot-sizing by a reseller who sells a perishable good. It was assumed that when it is economic to backlog demand, the reseller can plan for periods of shortage during which demand can be partially backordered. A solution procedure for solving the optimization problem was presented. **Benkherouf and Mahmond** (1996) presented an exact solution for the inventory replenishment problem with shortages, in which items are deteriorating at constant rate and demand rates increasing with time over a known and finite planning horizon. Replenishment was assumed to occur instantaneously at an infinite rate.

Giri, et al. (1996a) developed an EOQ model for deteriorating items where the demand rate, deterioration rate, holding cost and ordering cost are all expressed as linearly increasing continuous functions of time by allowing shortages which are completely backlogged. They assumed that planning horizon was finite and the replenishment periods are constant. **Giri, et al.** (1996b) discussed an inventory model with a inventory-level-dependent demand rate followed by a constant demand rate for items deteriorating at a constant rate, where the terminal condition of zero inventory of the end of the scheduling period has been relaxed. Sensitivity of the decision variables to change in the parameter values was also examined.

Bhunia and Maiti (1997) studied a deterministic inventory replenishment model for deteriorating items with time dependent demand over a finite planning horizon. They assumed that shortages are allowed and completely backlogged and the successive replenishment cycle lengths are in arithmetic progression. The model without shortages was also studied. Results were supported with numerical examples.

Chakrabarti and Chaudhuri (1997) considered inventory replenishment problem over a finite time horizon for a deteriorating item with a linear trend in demand, equal replenishment cycle and shortage in every cycle. The reorder number, the interval between two successive reorders and the shortage intervals were determined in an optimal manner so as to minimize the average system cost. **Jamal, et al.** (1997) developed a model for an optimal ordering policy for deteriorating items with allowable shortage and permissible delay in payment having constant rate of demand, instantaneous replenishment and exponential deterioration. **Urban and Baker** (1997) studied a deterministic inventory model in which demand is a multivariate function of price, time and inventory level.

Perry (1997) studied a double band control policy of Brownian perishable inventory system. A diffusion approximation and the stock level as the amount of items arriving during age of the oldest item are considered. **Fujiwara, et al.** (1997) studied an optimal ordering and issuing policy for a two-stage inventory system for perishable products. **Chen** (1998) studied the dynamic programming model for inventory items with Weibull distributed deterioration. The demand rate was assumed to be time proportional, shortages were allowed and completely backordered and the effects of inflation and time values of money were taken into consideration.

Bhunia and Maiti (1999) presented an inventory model for deteriorating items over finite time horizon where the demand increases linearly with time by assuming that the successive replenishment cycle lengths are same. The replenishment cost per replenishment was taken to be linearly dependent on the lot-size of that replenishment shortages were allowed and fully backlogged. **Chang and Dye** (1999) developed an EOQ model for deteriorating items with partial backlogging and demand rate a continuous and monotonic function of time. The backlogging rate is considered to be a decreasing function of waiting time for the next replenishment and items deteriorate at a constant rate over a finite planning horizon. The replenishment rate is assumed to be infinite and each cycle starts with shortages.

Wu, et al. (1999) derived an EOQ model for inventory of item that deteriorates at a Weibull rate and demand rates is a ramp type function of time and replenishment rate is infinite. **Aggoun, et al.** (1999) studied a stochastic discrete time single product inventory model for perishable and aging items. Here, inventory levels reviewed periodically and units in stock assumed to have a maximum life time of M periods.

Hsu (2000) considered an economic lot size (ECS) model for perishable products where an inventory stocks deterioration rate and its carrying cost is each period depend on the age of the stock. A model with concave production and inventory cost functions was developed. It was assumed that production occurs instantaneously at the beginning production period.

Liao, et al. (2000) developed an inventory model for initial-stock–dependent consumption rate when a delay in payment is permissible and shortages are not allowed. The effect of the inflation rate, deterioration rate, initial-stock-dependent consumption rate and decay in payment were discussed. **Liang and Liu** (2000) developed a discrete time (s, S) perishable inventory model with geometric inter-demand items and batch demands. **Williams**

and Patuwo (2000) developed a perishable inventory model with positive order lead times. The necessary equations to determine the single period, periodic review, optimal incoming quantity for a single product with a useful lifetime of two periods, subjected to a known the order lead-time and a lost sales policy are derived.

Goyal and Giri (2001) presented a review of deteriorating inventory literature by classifying by the shelf-life characteristics of the inventoried goods. They have further sub-classifying the models on the basis of demand variation and various other conditions or constraints like the price discount, permissible delay in payments, instantaneous and time value of money. **Kumar and Pakkala** (2001) developed a stochastic inventory model for deteriorating items with random supply quantity. They considered a continuous review (s, S) inventory system with stochastic demand for deteriorating item and random replenishment quantity.

Kar and Bhunia (2001) considered a deterministic inventory model for deteriorating items in two shops under single management. Here, after replenishment fresh units are stocked and sold from primary shop with a profit, whereas the deteriorated units are continuously transferred to a secondary shop as and when they are deteriorated and are sole at a reduced price. **Wang** (2002) developed an inventory replenishment policy for deteriorating items with shortages and partial backlogging.

Khanra and Chaudhuri (2003) discussed the order-level inventory problem where the demand rate is a continuous quadratic function of time and a constant function of the on-hand inventory deteriorates per unit of time. The solution of the model was discussed both for infinite and finite limit horizon. The solution procedure was illustrated by a numerical example and sensitivity analysis was also carried out. **Ghosh and Chaudhuri** (2004) developed an inventory model for deteriorating items having an instantaneous supply, a quadratic time- varying demand and shortages in inventory which are completely backlogged deterioration rate was assumed to follow a two-parameter Weibull distribution.

Teng and Yang (2004) developed a lot-size model for deteriorating items with deterministic time, varying demand and unit variable purchasing cost. They assumed that deterioration rate is constant, replenishment is instantaneous and planning horizon of the inventory problem is finite. The model allows for shortages and partial backlogging which is decreasing function of the waiting time.

Venkat Subbaiah, et al. (2004) developed inventory models for perishable items having stock- dependent demand rate, assuming the life time of the item is random and follows a three parameter Weibull distribution and replenishment occurs instantaneously at an infinite rate. **Lee and Wu** (2004) derived the EOQ model for inventory of items the deteriorate of a mixtures of exponential distribution rate, assuming that the demand rate and holding cost are continuous function of time and that replenishment rate is infinite.

Dye and Ouyang (2005) developed an inventory model for deteriorating items with stock-dependent demand, permitting shortages and time-proportional backlogging rate which is considered as a decreasing function of the waiting time for the next replenishment. Constant rate of deterioration and infinite rate of replenishment were assumed.

Srinivasa Rao, et al. (2005) developed and analyzed an inventory model for deteriorating items with the assumption that life time of the commodity is random and follows a generalized Pareto distribution. They obtained the optimal ordering and pricing policies of the model variations on the demand rate when it depends on time and selling price. **Sivakumar and Arivarignan** (2005) developed a perishable inventory system with service facilities and negative customers. A continuous review perishable (s, S) inventory system with a service facility consisting of finite waiting time capacity and a single serve is considered.

Gabriel, et al. (2005) developed pricing policies for perishable products with demand substitution. The optimal pricing for a family of substitute perishable products with demand correlation is studied. **Mandal Biswajit et al.** (2005) developed an inventory model for ameliorating items with linear price-dependent demand. For profit maximization, an instantaneous replenishment inventory model for items with the combined effect of both amelioration and deterioration is derived.

Teng, et al. (2005a) developed a pricing and lot-sizing model for a retailer when supplier provides a permissible delay in payments by assuming that annual demand is a decreasing function of selling price and that the selling price is necessary higher than the purchase cost. It was assumed the deterioration rate is constant and replenishment is instantaneous.

You, P.S. (2005) studied the problem of determining the order size and optimal prices for perishable inventory system under the condition that demand is time and price dependent. **Hou** (2006) derived an inventory model for deteriorating items with stock-dependent consumption rate and shortages under inflation and time discounting over a finite planning horizon. He presented an algorithm to determine the optimal order quantity and the optimal interval of the total cost function.

Ouyang, et al. (2006) considered an EOQ model for a retailer for deteriorating items to determine its optimal shortages interval and replenishment cycle when the supplier offers a permissible delay in payments, shortages were allowed and partially backlogged. The backlogging rate is a function of the waiting time and the unit selling price is assumed to be larger than the unit purchase cost. **Pal, et al.** (2006) considered the problem of determining the lot size of a single deteriorating item with the demand rate dependent on displayed stock level, selling price of an item and frequency advertisement. Shortages were allowed and partially backlogged with a variable rate which depends on the deterioration of waiting time up to the arrival or next lot.

Wu, et al. (2006) considered the problem of determining the optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand when shortages are allowed and partially backlogged where the backlogging rate is dependent on the waiting time for the next replenishment. The necessary and sufficient conditions for the existence and uniqueness of the optimal solution were shown. Sensitivity analysis of the optimal solution was carried and numerical examples presented.

Manna and Chaudhuri (2006) developed an EOQ model for deteriorating items with demand rate as a ramp type function of time. It is assumed that the finite production rate is proportional to demand rate and deteriorating rate is time proportional. **Feng and Xioa** (2006) presented a model to integrate pricing and capacity allocation decisions for perishable products. It is assumed that the supplier sells same products to different micro-markets at distinct prices.

Alfarez (2007) considered the inventory policy for an item with a stock-level dependent demand rate and a shortage-time dependent holding cost. **Dye, et al.** (2007) developed a deterministic inventory model for deteriorating items where the demand and deterioration rates were assumed to be continuous and differentiable function of price and time respectively. Shortages were allowed and the unsatisfied demand was partially backlogged at a negative exponential rate with the waiting time.

Huang (2007) studied an EOQ model in which the supplier offers a partially permissible delay in payments when the order quantity is smaller than the predetermined quantity. The retailer’s inventory system was modeled as a cost minimization problem to determine the optimal inventory cycle time and optimal order quantity. **Roy and Chaudhuri** (2007) developed an order level inventory model for a deteriorating item, taking the demand to be dependent on the sale price of the item and incorporating the concept of the special sale campaign by way of price reduction into the model.

Srinivasa Rao, et al. (2007) developed and analyzed an inventory model with the assumption that the lifetime of the commodity is random and follows a generalized Pareto distribution. They assumed the replenishment is instantaneous, the demand is a function of stock and the money value is subject to inflation. By minimizing the total cost function they obtained the optimal ordering quantity and cycle length. Results were demonstrated using numerical examples. **Sen and Chakrabarty** (2007) considered an order-level inventory model with shortages having variable rate of deterioration and alternating replenishment rates. It is assumed that the items with Weibull decay are produced during sub periods to meet the change in demand pattern and market fluctuations.

Manjusri and Sudipta (2007) developed ordering policy for deteriorating items with two-component demand and price break with shortages. The EOQ model considered has varying demand rates: (i) for certain period the demand is quadratic depend on stock (ii) the rest of the period the demand rate is constant. **Dye, et al.** (2007) developed a deterministic inventory model for deteriorating items with capacity constraint and time-proportional backlogging rate.

Chen and Chen (2008) studied a model to determine replenishment schedule and selling price for a monopolistic retailer who stocks a single product that is subject to continuous decay, having a price-dependent and time-varying demand, and has the objective to maximize the total profit stream over multi-period planning horizon. **Madhavi, et al.** (2008) developed an inventory model with the assumption that lifetime of the item is random and follows two parameter exponential distribution. Keeping the deteriorated items also in the inventory for seconds sale. It was assumed that the deteriorated item is not thrown out as a waste but it is sole for certain price discount. Demand rate was assumed to be a function of selling price.

Panda, et al. (2008) studied a single item order level inventory model without shortages for a seasonal product where the demand rate is represented by a ramp-type dependent function. They assumed that replenishment rate is infinite and a constant fraction of on hand inventory deteriorates per unit time. A numerical example was presented and sensitivity analysis of the model was carried out. **Roy** (2008) developed an EOQ model with and without shortages for deteriorating items, where deterioration rate and holding cost are expressed as linearly increasing functions of time, demand rate is a function of selling price and replenishment rate is infinite.

Sana and Chaudhuri (2008) considered an EOQ model without shortages for various types of deterministic demand when delay in payment is permitted by retailer to supplier and the supplier offers discount rates of price at different delay periods. Replenishment was assumed to be instantaneous. **Soni and Shah** (2008) studied a model for inventory system with stock dependent demand and infinite replenishment when the supplier offer two progressive credit periods to the retailer to settle his account. The objective function to be optimized is considered as the total cost of an inventory system. The effect of parameters on the objective function is studied numerically.

Tsao and Sheen (2008) developed a finite time horizon inventory model for deteriorating items with price and time dependent market demand under permissible delay in payments. The objective was to determine the optimal retail price, promotional effort and replenishment quantities throughout a multi-cycle planning horizon so that the net profit is maximized. Replenishment was assumed to occur instantaneously. **Hsieh, et al.** (2008) developed a deterministic inventory model for deteriorating items with two-warehouses by minimizing the net present value of the total cost by permitting shortages and backlogging.

Rong, et al. (2008) discussed an inventory policy for a deteriorating item with imprecise lead-time, partially/fully backlogged shortages and price dependent demand is developed under two-warehouse system. **Arya, et al.** (2009) developed inventory models for single deteriorating item with variable deterioration rate depending on time and demand depending on stock. Shortages were allowed and partially backlogged where the backlogging rate is dependent on the deterioration of the waiting time up to the arrival of next lot. The replenishment was assumed to occur instantaneously at an infinite rate.

Mahata and Goswami (2009a) developed an infinite time horizon fuzzy EOQ model for deteriorating items with stock dependent demand rate and non-linear holding cost by taking deterioration rate is a triangular fuzzy number over an infinite time horizon. Replenishment was assumed to be instantaneous. **Mahata and Goswami** (2009b) formulated inventory model with imprecision inventory costs for deteriorating items under inflation. Shortages were allowed and the demand rate taken as a ramp type function of time. Numerical example is given to illustrate the problem. Sensitivity analysis was also carried to indentify the most sensitive parameters in the system.

Maiti, et al. (2009) developed inventory model for an item in stochastic environment with price-dependent demand over a finite time horizon considering probabilistic led-time and allowing shortages. Assuming the rate of replenishment is infinite and unit price is inversely related with the advance payment amount, mathematical expression for the expected average profit of the system was derived.

Ouyang, et al. (2009) developed an EOQ model under the conditions of permissible delay in payment by considering the following situations simultaneously: (i) the retailer’s selling price per unit is higher than the unit purchase price, (ii) the interest rate charged by a bank is not necessarily higher than the retailer's investment return rate, (iii) selling items deteriorate continuously of a constant rate, and (iv) the supplier may offer a partial permissible delay in payments even if the order quantity is less than a specified amount. **Panda, et al.** (2009a) studied a single item economic order quantity model in which the demand is stock dependent up to time T from time of fresh replenishment beyond which it is constant. They investigated how much discount on selling price may be given during deterioration to maximize the profit per unit time and whether a pre-deterioration discount affects the unit profit or not.

Panda, et al. (2009c) developed a perishable inventory model with time dependent quadratic ramp-type and partial backlogging. It was assumed that the deterioration of inventory starts after a certain time. They derived the profit function and unique existence of the solution was established. A numerical example was presented and sensitivity of the model was also examined.

Roy, et al. (2009) developed an inventory for deteriorating item with linearly displayed stock dependent demand in imprecise environment (involving both fuzzy and random parameters) under inflation and time value of money. It was assumed that the on hand inventory deteriorates at a constant rate. **Shah and Poonam** (2009) developed a mathematical model without shortages to obtain optimal ordering policy of time dependent deteriorating item when demand rate is dependent on displaying stock level and frequency of advertisement through media.

Singh, et al. (2009) developed a determining inventory model with power demand patterns in which inventory is depleted not only by demand but also by deterioration at the rate which is assumed to be time dependent with the concept of lifetime of items. **Skouri, et al.** (2009) developed an order level inventory model with ramp type demand rate, Weibull deterioration rate, instantaneous replenishment and partial backlogging rate is any non increasing function of the waiting time up to the next replenishment.

Srinivasa Rao, et al. (2009) developed an inventory model for deteriorating items having additive exponential life time and selling price dependent demand rate by assuming instantaneous replenishment. **Uthayakumar and Geetha** (2009) investigated an instantaneous inventory model for non- instantaneous deteriorating items under inflation and time discounting over a finite planning horizon where the consumption rate is stock-dependent and shortage are allowed and partially backlogged.

Balki (2010) developed a finite horizon trade credit economic ordering policy for an inventory model with deteriorating items under inflation and time value of money when shortages are not allowed. The demand and the deterioration rates were assumed as continuous and known functions of time. **Banerjee and Sharma** (2010) studied a deterministic inventory model where product under consideration has price and time dependent seasons demand rate. They considered the case where order for replenishment is placed at the end of the season in which inventory depletes completely and inventory once order can be used for more than one season.

Chang, et al. (2010) dealt with the problem of determining the optimal selling price and order quantity simultaneously under EOQ models for deteriorating items when the demand is a function of the on display stock level and the selling price per unit. Imposing a limited maximum amount of stock displayed in a supermarket, they formulated two types of mathematical models to manifest the extended EOQ models for maximizing profits and derived the algorithms to find the optimal solution. **Begum, et al.** (2010) developed an instantaneous replenishment and profit maximization policy for deteriorating items with selling price dependent demand and stochastic deterioration shortages were allowed and completely backlogged.

Chang and Lin (2010) derived a partial backlogging inventory model for non-instantaneous deteriorating items with stock-dependent demand rate over a finite panning horizon by considering the effects of inflation and time value of money. They assumed that replenishment rate is infinite and a constant fraction of the on-hand inventory deteriorates per unit of time. **Goswami, et al.** (2010) investigated the retailer’s inventory system for deteriorating items as a cost minimization problem to determine the retailer’s optimal inventory policy. They assumed that the supplier would offer the retailer a delay period and the retailer also adopted the trade credit policy to stimulate his/her customer demand.

Hsieh and Dye (2010) developed an inventory lot size model for deteriorating items under inflation using a discounted cash flow (DCF) approach over a finite planning horizon allowing a multivariate demand function of price and time and partial backlogging. It assumed that a constant fraction of the on-hand inventory deteriorates per unit of time. **Khanra, et al.** (2010) studied a single-item economic order quantity model with stock displaying and price dependent demand rate that maximizes expected average profit over an infinite planning horizon. The sensitivity of the optimal solution to changes in the values of different parameters was also examined.

Liao and Huang (2010) studied a two warehouse inventory model with instantaneous replenishment and constant demand for optimizing the replenishment cycle time for a single deteriorating under a permissible delay in payments. **Mandal** (2010) derived an EOQ inventory model for items deteriorating at a Weibull distribution rate assuming that the demand rate is infinite results were illustrated with a numerical example and sensitivity analysis was carried.

Mishra and Singh (2010) developed a deterministic inventory model for items deteriorating at a constant rate in which shortages are allowed and partially backlogged. It was assumed that replenishment rate is infinite and demand rate is linear function of time. The backlogging rate is assumed as variable and dependent on the length of the next replenishment. **Patra** (2010) developed an order level inventory model, continuous in unit and time for deteriorating items under deterministic demand and instantaneous delivery for one and two warehouse inventory problems, shortages allowed and partially backlogged.

Roy and Chaudhuri (2010) developed an inventory model for perishable item over a finite planning horizon assuming that the demand rate depends on time and selling price of the item. The deterioration rate of the item is taken to be constant and time-value of money is also considered shortages in inventory were allowed and completely backlogged. **Sana** (2010) dealt with on EOQ model over a infinite time horizon for perishable items where demand is price dependent and partial backorder is permitted. The rate of deterioration was taken as time proportional and it was assumed that replenishment rate is instantaneously infinite and shortage occurs and starting of the inventory cycle. **Sarker, et al.** (2010) considered an economic order quantity model for various types of deterministic demand patterns in which the delay periods and different discounts rates on purchasing are offered by the supplier to the retailer in the presence of inflation.

Ruxian Li, et al. (2010) reviewed the recent studies in deteriorating inventory and proposed some key factors which should be a considered in the deteriorating inventory studies such as demand, deteriorating rate and other factors like price discount, money and so on. They distinguished the current literature in two categories: the studies based on an enterprise and those based on supply chain.

Shah and Mishra (2010) developed an EOQ model without shortages when units in inventory deteriorate at a constant rate, salvage value is associated to deteriorated units and demand is linearly stock dependent. The sensitivity analysis was also carried out to analyze the effect of critical parameters. **Tripathy and Mishra** (2010) developed an EOQ model for deteriorating items where deterioration rate follows two-parameter Weibull distribution and holding costs are expressed as linearly increasing functions of time. Replenishment rate is instantaneous and demand rate is considered to be a function of selling price.

Yang, et al. (2010) developed an economic order quantity (EOQ) model, in which (1) shortages are partially backlogged, (2) the effects of inflation and time value of money are considered, and (3) the replenishment cycles and the shortages intervals are time varying. **Villiathal and Uthayakumar** (2011) studied the optimal pricing and replenishment policies of an economic order quantity model for determining items with partial backlogging over an infinite time horizon. They studied on EOQ model under the replenishment policy starting with no shortages. The backlogging rate was taken to be any non- increasing function of the waiting time up to the next replenishment.

Hung (2011) developed an order level inventory model for arbitrary demand rate and arbitrary deterioration rate in the consideration of partial backorder. **Liang and Zhou** (2011) considered a two warehouse inventory model for deteriorating items with constant demand and infinite replenishment rate under conditionally permissible delay in payments. **Misra, et al** (2011) derived an optimal inventory model with a permissible delay in payment under inflation over finite planning horizon. Instantaneous replenishment and constant demand rates were assumed.

Sana (2011) developed two finite time horizon deterministic EOQ models by assuming infinite rate of replenishment and constant rate of deterioration. Prices at different periods were considered as decision variable. The objective was to find the optimal ordering quantity and optimal sales prices that maximizes the vendor's total profit. The results were discussed with numerical examples. **Yang, et al.** (2011) studied a deterministic inventory model for deteriorating items with stock-dependent demand and limited storage space in which the terminal condition of zero ending inventories is relaxed. Replenishment was assumed to occur at an infinite rate and shortages were allowed and partially backlogged.

Biswajit Sarkar (2012) developed an EOQ model with delay in payments and stock dependent demand in the presence of imperfect production. A model is considered to investigate the retailer's optimal replenishment policy under permissible delay in payment with stock dependent demand within the EOQ (Economic Order Quantity) framework. Numerical examples and sensitivity analysis are also presented.

Chandra K. Jaggi, et al. (2012) studied the fuzzification of EOQ Model under the condition of permissible delay in payments. They formulated an economic order quantity inventory model under the condition of permissible delay in payments in fuzzy environment. The parameters of the model, such as permissible delay period and cycle length, are taken to be trapezoidal Fuzzy numbers. The cost function has been defuzzified using signed distance method. A numerical example is presented.

1.3 REVIEW ON PRODUCTION LEVEL INVENTORY MODELS FOR DETERIORATING ITEMS WITH RANDOM LIFE TIME:

In this section, we briefly review some of the contributions in Economic production quantity models for deteriorating items. Here we consider the production (replenishment) rate is not instantaneous. Several authors developed various EPQ models with various assumptions on demand, production (replenishment) and life time of the commodity. There does not exist a unique model which suits for all situations since the practical considerations and constraints governing the process are wide in nature.

Mak (1982) studied a production lot size model which incorporates an unfilled-order backlog for an inventory system with exponential decaying items. Approximate expressions were obtained for the optimum production lot size, the production cycle time and the total cycle time. Numerical example was given. **Mukharjee and Pal** (1986) considered an order level production inventory policy for items subject to a general rate of deterioration where demand rate is constant and replenishment is finite.

Hwang and Hwang (1982) studied an optimal issuing policy is a production lot size system for items with Weibull distribution. **Chowdhary and Chaudhuri** (1983) formulated an inventory model for deteriorating items with finite rate of production and constant rate of deterioration both deterministic and probabilistic cases. **Mandal and Phaujdar** (1989) developed a single item stock-control model with shortages for deteriorating items having uniform rate of production and variable demand rate dependent an instantaneous inventory level.

Bhunia and Maiti (1998) studied an EPQ model for deteriorating items with and without shortages. The rates of demand and deterioration of any moment of time was taken to be a linearly increasing function of time. Results were illustrated with numerical examples. **Yan and Cheng** (1998) studied a perishable single item production inventory model which considers the rates of production, demand and deterioration as time dependent functions by allowing demand shortages which are partially backlogged. They discussed the optimal production stopping and restarting times which minimize the total relevant cost per unit.

Su, et al. (1999) developed a deterministic production inventory model for deteriorating items with an exponential declining demand over a fixed time horizon and production rate dependent demand. **Balkhi** (2001) considered a production lot-size inventory model for deteriorating items over a finite planning horizon where the demand, production and deteriorating rates were assumed to be known and continuous functions of time. Shortages were allowed and completely backlogged. The conditions under which the system total cost attains its (unique) global minimum were also derived.

Sujit and Goswami (2001) considered a replenishment policy for items with finite production rate and fuzzy deterioration rate. The deterioration rate is considered constant to solve economic production quantity (EPQ) model. The effect on loss in production quantity due to aged machine is also studied. **John Mathew** (2002) developed and analyzed inventory models for perishable items with Weibull rate of decay and finite replenishment rate as demand rate varies with selling price and time. He also analyzed the case when the lifetime of the commodity is random and follows a mixture of Weibull distribution, finite replenishment rate and demand is a function both selling price and time. In all cases, models with and without shortages were studied and sensitivity analysis were studied.

Goyal and Giri (2003) considered the production-inventory problem in which the demand, production and deterioration rates of a product are assumed to vary with time. They developed two models for the problem by employing different modeling approaches over an infinite planning horizon; shortages were allowed and partially backlogged. **Samata and Roy** (2004) developed a continuous production control inventory model items deteriorating exponentially with time and in which two different rates of production are available and it is possible that production started at one rate and after some time it may be switched out to another rate. Demand rate was assumed to be constant and shortages were allowed.

Sana, et al. (2004) developed a production inventory model for deteriorating item over a finite planning horizon with a time-varying demand, finite production rate and shortages. The optimal number of production cycles that minimizes the system cost was determined and sensitivity analysis is carried. **Teng and Chang** (2005) studied an economic production quantity (EPQ) model for deteriorating items when the demand rate depends on stock –level and selling price per unit by imposing a ceiling on the number of on-display stocks and the deteriorating rate was constant.

Teng, et al. (2005c) studied a deterministic economic production quantity model by considering time varying cost. They proved that the optimal production schedule uniquely exists and showed that the total cost is a convex function of the number of replenishments. **Hsieh and Lee** (2005) studied two economic manufacturing quantities (EMQ) models with un-repairable and repairable standby key modules that determine the economic production process, where the key module of the production unit deteriorates over time and incurs some portion of defective items.

Sivakumar and Arivarignan (2005a) developed an inventory model with multiple rates of production during sub-periods for the constraints that arise due to change in demand pattern, market fluctuation etc., **Lin and Gong** (2006) studied a production inventory system of deteriorating item subject to random machine breakdowns with a fixed repair time. **Jolai, et al.** (2006) considered a production model for perishable item that follows a two parameter Weibull distribution. They presented an optimization frame work to derive optimal production over a fixed planning horizon for items with a stock dependent demand rate under inflationary conditions. Shortages were allowed and partially backlogged at a fixed rate.

Liao (2007) derived a production model for the lot-size inventory system with finite production rate, taking into consideration the effect of decay and the condition of permissible delay in payments. **Srinivasa Rao and Begum** (2007) developed and analyzed an inventory model for deteriorating items with generalized Pareto decay and selling price dependent demand with finite replenishment.

Kar, et al. (2008) discussed multi-objective inventory model for deteriorating items with space constraint in a fuzzy environment. It is brought out that the machine production rate is not always constant but may be treated as a decision variable. **Darwish** (2008) developed two models (with and without shortages) by considering a relationship between the setup cost and he production run length.

Sugapriya and Jeyaraman (2008) derived a production cycle time for an EPQ model with non-instantaneous deteriorating items allowing price discount using permissible delay in payments. **Chen** (2008) discussed economic production run length and warranty period for products with Weibull lifetime. **Sugapriya and Jeyaraman** (2008a, 2008b) discussed EPQ model for a single product subject to non-instantaneous deterioration following exponential distribution where production and demand rates were assumed to be constant and shortages are not allowed using permissible delay in payments and in the second model holding cost is assumed to vary with time.

Mirzazadeh, et al. (2009) discussed an inventory model under uncertain inflationary conditions, finite production rate and inflation-dependent demand rate for deteriorating items with shortages. **Lee and Hsu** (2009) developed a two-warehouse inventory model for deteriorating items with time-dependent demand. The variation in production cycle times to determine the number of production cycles and the time for replenishment during a finite planning horizon is considered. **Panda, et al.** (2009b) developed a single item production inventory model for deteriorating items to determine optimal production stopping time in the first production cycle for a few products introduced in the market.

Das, et al. (2010) presented a production lot-size inventory model in which the production rate constitutes of productions during both regular time and overtime. The demand rate was assumed as stock-dependent and the stock itself is depleted due to demand and deterioration. **Hu and Liu** (2010) investigated the optimal replenishment policy under conditions of permissible delay in payments and allowable shortages within the economic production quality (EPQ) framework. They assumed that the replenishment rate is finite and the unit selling price is not necessarily equal to the unit purchasing price.

Manna and Chiang (2010) developed two deterministic economic production quantity (EPQ) models with and without shortages for deteriorating items with demand rate as a ramp type function of time. It was assumed that the finite production rate is proportional to the time-dependent demand rate and the unit production cost was inversely proportional to the production rate. Results were illustrated with the help of numerical example.

Sridivi, et al. (2010) developed and analyzed an inventory model with the assumption that the rate of production is random and follows a Weibull distribution and the demand is a function of selling price. By maximizing the profit rate function they obtained the optimal ordering and pricing policies of the model. The sensitivity of the model with respect to the parameters and costs was also analyzed. **Tripathy, et al**. (2010) developing an EPQ model with no shortages for items with time varying holding cost, linear deterioration rate and constant production and demand rates. Partially deteriorated items were allowed to float into the market with a discount.

Uma Maheswara Rao, et al. (2010) developed and analyzed an inventory model for production process with the assumption that the lifetime of the product is random and follows Weibull rate of decay. Assuming that the demand is a function of stock on hand and shortages are allowed they derived optimal ordering quality. The sensitivity of model with respect to the parameters and costs was also studied. **Valliathal and Uthayakumar** (2010a) discussed a deterministic finite horizon economic production lot size model for deteriorating items under permissible delay in payments. Shortages were allowed and partially backlogged.

Goswami, et al. (2010) studied optimal retailer replenishment decisions for deteriorating items under two level of trade credit policy within economic production quality frame work. **Pentico and Drake** (2011) reviewed deterministic EOQ and EPQ models. They presented various methods adopted in inventory models and extension that add other considerations, such as pricing perishable or deteriorating inventory, time-varying or stock dependent demand quality discounts, or multiple warehouses. **Dong and Jiang** (2011) studied the limiting distribution of inventory level of a perishable inventory model. In their model they assumed that each perishable item has finite lifetime, and only one item is consumed each time. They used the backward equations and limit distribution of Markov processes.

Jolai, et al. (2011) studied the inventory control of perishable items in a two-echelon Supply Chain. They developed the inventory model with random lifetime. They also considered that the productions rate is constant. Deterioration in this stage is modeled using Weibull distribution. The behavior of the model is analyzed by using numerical studies. **Vinod Kumar Mishra and Lal Sahab Singh** (2011) developed a production inventory model for time dependent deteriorating items with production disruptions. They considered the storage item is deteriorated during normal and disrupted production periods in which shortages are not allowed and demand rate is deterministic and constant. The model is solved analytically to determine the optimal production time during normal and disrupted production periods.

Sankar and Moon (2011) considered a finite time horizon production inventory model for stochastic demand with shortages and the effect of inflation. The defective item was assumed to follow a Weibull distribution. They derived profit function by using distribution of demand. Production rate of the inventory system is considered to be constant.

Tsai (2012) developed an optimal ordering and production policy for a recoverable item inventory system with learning effect. They determined the optimal integrated economic order quantity and economic production quantity policy in a recoverable manufacturing environment. The total cost functions of the models are derived. Search procedures are used to determine optimal policy parameters. Numerical examples are provided.

Su (2012) developed an optimal replenishment policy for an integrated inventory system with defective items and allowable shortage under trade credit. They considered a single-supplier, single-retailer integrated inventory model that accounts for defective items that arrive in a retailer's order under a full-lot inspection policy.

Chang, et al. (2012) developed an economic manufacturing quantity model for a two-stage assembly system with imperfect processes and variable production rate. They considered a two-stage assembly system with imperfect processes. The former is an automatic stage in which the required components are manufactured. The latter is a manual stage which deals with taking the components to assemble the end product. Shortage is allowed, and the unsatisfied demand is completely backlogged. An algorithm for the computations of the optimal solutions under the constraint of assembly rate is given. A numerical example and sensitivity analysis are presented.

1.4 FOCUS OF THE PROJECT:

Inventory control plays a dominant role in many practical situations at places such as production processes, manufacturing units, transportation, market yards, ware houses, assembly lines etc. Inventory models provide the basic frame work for analyzing several production systems. The inventory models are broadly categorized into two groups namely, (i) economic order quantity models (EOQ models) and (ii) economic production models (EPQ models).

EPQ models can be applied to a system where a product is produced on a production line in order to meet the demand or the product is procured and stored for supply of goods to meet demand. The EPQ models are more common in production and manufacturing processes, warehouses, etc. In EPQ models the major string is relaxing some of the assumptions regarding production (replenishment), nature of the commodity and demand pattern (**Osteryoung, et al.** (1986)).

Recently much emphasis was given for analyzing EPQ models for perishable items. Deterioration is a natural phenomenon of several commodities. The deterioration is highly influenced by several random factors like storage facility, temperature, environmental conditions, quality of raw material etc. Several author various EPQ models for deteriorating items with various assumptions on lifetime of commodity.

Nahimias (1982), Raafat (1991), Goyal and Giri (2001), Ruxian Lie, et al. (2010) and Pentico and Drake (2011) have reviewed the literature on inventory model for deteriorating items. To develop the EPQ models it is needed to ascribe a probability distribution to the lifetime of commodity. Ghare and Schrader (1963), Shah and Jaiswal (1977), Cohen (1977), Aggarwal (1978), Dave and Shah (1982), Pal (1990), Kalpakam and Sapna (1996), Giri and Chaudhuri (1999) assumed that the lifetime of commodity follows an exponential distribution. Tadikamalla (1978) assumed gamma distribution to the lifetime of commodity, Covert and Philip (1973), Philip (1974), Goel and Aggarwal (1980), Venkata Subbaiah, et al. (2004) assumed Weibull distribution to the lifetime of commodity. Nirupama Devi, et al. (2001) developed inventory models with mixture of Weibull distribution for the lifetime of commodity, Srinivasa Rao, et al. (2005) developed inventory model with generalized Pareto lifetime, Xu and Li (2006) developed a two-warehouse inventory model for deteriorating items with time-dependent demand, Rong, et al. (2008) studied a two-warehouse inventory model for deteriorating items with partially/fully backlogged shortages and fuzzy lead time, Srinivasa Rao, et al. (2009) studied an inventory model for deteriorating items having additive exponential life time and selling price dependent demand rate, Chang and Lin (2010) studied an inventory model for deteriorating items with stock dependent demand, Biswajit Sarkar (2012) developed an EOQ model with delay in payments and stock dependent demand in presence of imperfect production. However, in all these EOQ models, it is assumed that the replenishment is instantaneous with finite rate.

Mukarjee and Pal (1986), Sujit and Goswami (2001), Goyal and Giri (2003) developed inventory models with finite rate of replenishment (production). Panda and Chatarjee (1987), Mandal and Phajudar (1989) and Sana, et al. (2004) developed inventory models with uniform rate of replenishment (production). Perumal and Arivarignan (2002) considered two rates of production in an inventory model. Pal and Mandal (1997) and Sen Chakrabarty (2007) developed alternating replenishment rates. Lin, et al. (2006), Maiti, et al. (2007), Hu and Liu (2010), Uma Maheswara Rao, et al. (2010) have developed inventory models for deteriorating items with constant rate of production (replenishment). Venkata Subbaiah, et al. (2011) have developed EPQ model with alternating rate of replenishment, Essay, et al. (2012) have developed EPQ models with stock dependent production and Weibull decay.

However, in many production lot size models the replenishment rate is not constant or uniform and will have a variable rate of production (replenishment), since the production or replenishment is influenced by several random factors like transportation, quality of raw materials, availability, packaging, environmental conditions etc. For example in case of seafood’s and agricultural products the uncertainty in the yield effects the replenishment. Also it can be observed several production processes dealing with perishable items will have a variable rate of replenishment. For modeling this sort of situation it is needed to consider the replenishment (production) is random and follows a probability distribution. Very little work has been reported in literature regarding economic production quantity models with random production except the works of Sridevi, et al. (2010) and Srinivasa Rao, et al. (2010) and , Srinivasa Rao and Lakshmana Rao (2014) who have develop and analyzed inventory models for deteriorating items with random replenishment. They assumed that the deterioration rate is constant.

Since several of the items are having variable rate of decay and the decay starts only after certain period of time it is reasonable to consider that lifetime of commodity is random and follows Pareto distribution. The Pareto distribution includes uniform distributions as limiting cases. With this motivation in this project we develop and analyze some EPQ models for deteriorating items with Exponential replenishment (production) and Pareto decay having various patterns of demand.

First we develop and analyze an inventory model for deteriorating items with the assumption that the replenishment (production) is random and follows a Exponential distribution. It is further assumed that the lifetime of the commodity is random and follows a Pareto distribution. It is also assumed that the demand rate is a function of selling price and is of the form $f(s) = a - bs, \quad a > 0, b < a$, where 'a', 'b' are constants, 's' is the selling price. Assuming shortages are allowed and fully backlogged the inventory model is derived. This model is extended to the case of without shortages.

Another variation in this model is also investigated by considering that the demand is a function of time. It is a common belief that time has a tremendous influence on the demand of the commodity. Here, it is assumed that the demand follows power pattern with indexing parameter. For different values of indexing parameter it includes different types of demand. Assuming the shortages are allowed and fully backlogged, the inventory model with Exponential rate of production and Pareto decay having time dependent demand is analyzed. This model is extended to the case of without shortages.

Both selling price and time put together influence the demand in some commodities like edible oils and agricultural products. Hence another economic production quantity model with Exponential rate of production and Pareto decay having time dependent demand is developed and analyzed for with and without shortages. Here, the demand rate is a function of time. It is of the form $f(t) = \frac{dt^{\frac{1}{n}-1}}{n.T^{\frac{1}{n}}}$ where 'n' is indexing parameter and 'T' is the cycle length. This demand function includes different types of demand.

In all these models, using the differential equations the instantaneous state of inventory at time 't', the stock loss due deterioration, production (order) quantity are derived. With suitable cost considerations, the total cost function per unit time and the profit rate function are derived. By optimizing the total cost function or profit rate function the optimal production schedules and ordering quantities are derived. The sensitivity of the model with respect to parameters and costs are also analyzed. Through numerical studies, the solution procedures for optimal policies are demonstrated. These models also include some of the earlier models as particular cases for specific or limiting values of the parameters. The models derived in this project are having practical utilization in seafood's industries, food and vegetable markets, cement, chemical industries warehouses and market yards etc., where the commodity lifetime is random and follows Pareto distribution and the production is governed by Exponential distribution.

1.5 EXPONENTIAL DISTRIBUTION:

In this section we briefly describe Exponential distribution and its properties. Here, it used to characterize the production (replenishment) of the systems studied in this project. We assume that replenishment (production) is random and follows an Exponential distribution with the probability density function of the form (Johnson Kotz and Balakrishanan (1994))

$$f(t) = \lambda e^{-\lambda t}, \lambda > 0, t > 0 \qquad (1.5.1)$$

The distribution function is $\mathrm{F(t)} = 1 - e^{-\lambda t}$ (1.5.2)

The survival function is $G(t) = e^{-\lambda t}$ (1.5.3)

The instantaneous rate of replenishment is

$$k(t) = \frac{f(t)}{1-F(t)} = \frac{f(t)}{G(t)} = \lambda \qquad (1.5.4)$$

Its mean is $\frac{1}{\lambda}$ (1.5.5)

Its variance is $\frac{1}{\lambda^2}$ (1.5.6)

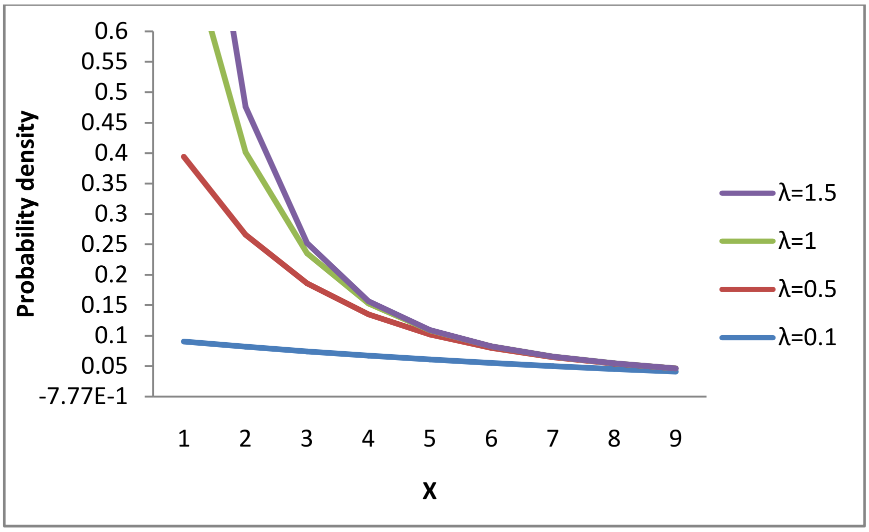


Fig 1.1: Probability density function of Exponential distribution for different values of

1.6 PARETO DISTRIBUTION:

In this section, we briefly discuss some distributional properties of Pareto distribution which is used as life time distribution of the commodities. Assume that the life time of the commodity follows a Pareto distribution having the probability density function of the form (Johnson Kotz and Balakrishnan (1994)).

$$f(t) = \frac{\alpha\beta^{\alpha}}{t^{\alpha+1}}, \alpha > 0, 0 < \beta < 1 \tag{1.6.1}$$

where α is the shape parameter and β is scale parameter

The mean life time of the commodity, μ is

$$\mu = \frac{\alpha\beta}{\alpha-1} \tag{1.6.2}$$

Its distribution function is $F(t) = 1 - \left(\frac{\beta}{t}\right)^{\alpha}$ (1.6.3)

The instantaneous rate of deterioration/hazard rate of the on hand inventory is

$$h(t) = \frac{f(t)}{1-F(t)} = \frac{\alpha}{t}, t > 0 \tag{1.6.4}$$

The different shapes of the frequency curve and hazard rate for various values of the parameters are shown in Figure 1.2, Figure 1.3 and respectively.

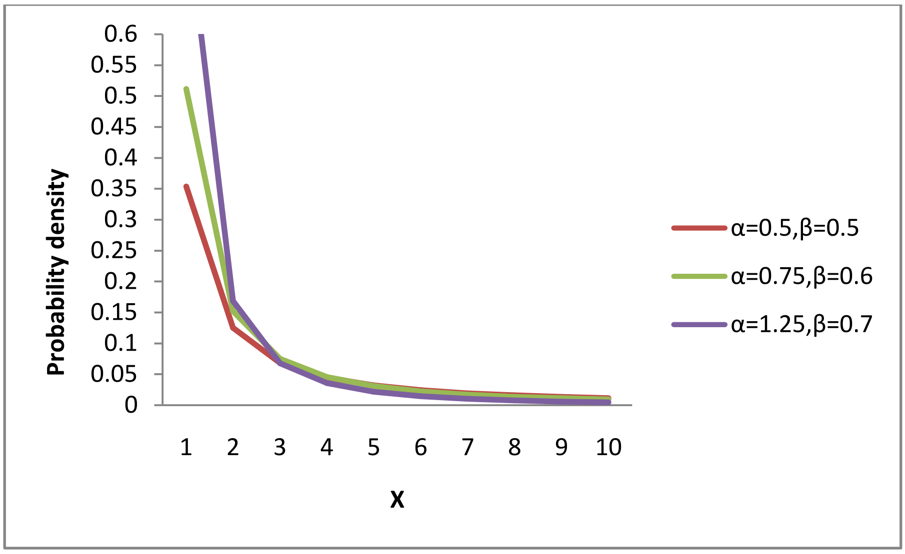


Fig 1.2: Probability density function of the Pareto distribution for different values ‘α’ and’ β’

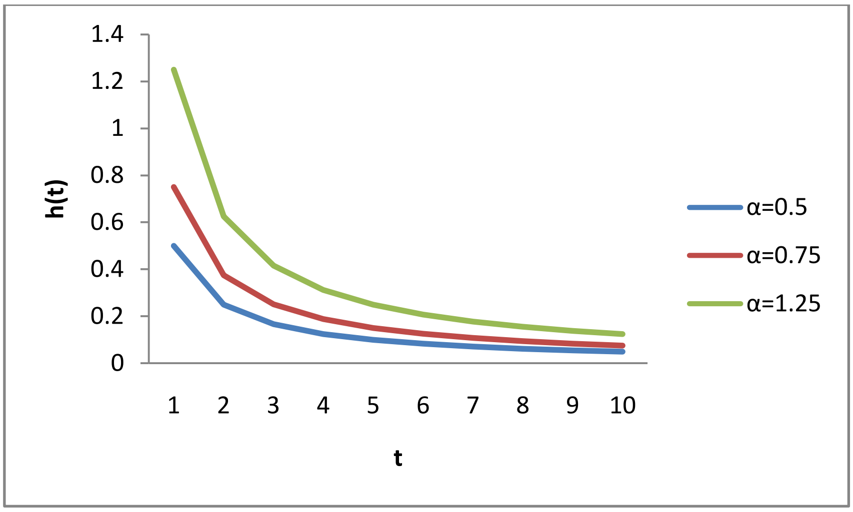


Fig 1.3: Hazard rate curve of h (t) for different values of ‘α’

1.7 ORGANISATION OF THE PROJECT:

This project is divided into four chapters. The chapter wise outline of the project is briefly presented as follows.

In **Chapter I**, a brief discussion on inventory models for deteriorating item is presented. Motivation of the present work is given in the focus of the project. The review of literature on EOQ models for deteriorating items and EPQ models for deteriorating items are presented. The basic characteristics of the probability distributions used in the project are also given as ready reference. The organization of the project is also presented.

In **Chapter II**, an inventory model for deteriorating items with Exponential replenishment and Pareto decay having selling price dependent demand is developed and analyzed. Using differential equations the instantaneous state of inventory is derived. With suitable cost considerations the Profit rate function is obtained. By maximizing the Profit rate function the optimal pricing and ordering policies are obtained. Results are illustrated numerically and sensitivity analysis is presented.

In **Chapter III**, an inventory model for deteriorating items with Exponential replenishment and Pareto decay having time depend demand is discussed. Here, it is assumed that demand is a power function of time and follows power pattern with an indexing parameter, the optimal ordering policies of the model are obtained. The sensitivity of the model with respect to the parameters and costs is also studied.

In **Chapter IV**, the results derived in the earlier chapters are summarized with conclusions. The scope for further work in this area of research is also pointed.

CHAPTER – II

INVENTORY MODEL FOR DETERIORATING ITEMS WITH EXPONENTIAL REPLENISHMENT AND PARETO DECAY HAVING SELLING PRICE DEPENDENT DEMAND

INVENTORY MODEL FOR DETERIORATING ITEMS WITH EXPONENTIAL REPLENISHMENT AND PARETO DECAY HAVING SELLING PRICE DEPENDENT DEMAND

2.1 INTRODUCTION:

Inventory models generate more interest due to their ready use at different locations such as transport systems, production processes, warehouses and market yards, etc., A number of models of inventory were developed and analyzed in order to study different stock structures. The essence of the product, demand and replenishment are very important factors that have an effect on the supply systems. In many inventories systems assumed the replenishment is limitless, and these models of inventory are instantaneous, in addition that replenishment rate is fixed and finite.

Deb and Chaudhuri (1986) analyzed the finite rate of production. Bhunia and Maiti (1997) developed two models; production is considered to be a function of the stock level in one system and output is a function of the demand rate in another. Billington (1987) studied the EOQ model was tested with no backorders. Rein, et al. (2000) studied a stochastic stock model of two discrete systems of output. Sen and Chakrabarthy (2007) analyzed stock order level model with adjustable deterioration frequency and alternative rate of replenishment. Shaibaji Panda and Nilunja Mohan Modak (2015) and Sridevi, et al. (2010) developed inventory models assuming the output is random and follows the distribution of Weibull. They assumed that the commodity's lifespan is random and follows an exponential distribution with a constant deterioration frequency. Srinivasa Rao and Lakshmana Rao (2014) considered inventory model assuming replenishment is random and follows the distribution of Weibull and commodity lifetime is random and follows generalized distribution of Pareto with variable deterioration rate. They assumed that replenishment is random and the commodity life is random with constant rate of deterioration except Srinivasa Rao and Lakshmana Rao (2014).

Furthermore, in many other practical situations, such as the food processing industry, product life varies depending on selling price. It's also known that the commodity's lifespan has a finite upper limit and the rate of decay is proportionate to the time. This nature could well be characterized by the distribution of Pareto. This nature could well be characterized by the distribution of Pareto. The distribution Pareto is capable of representing the lifetime of a commodity with a variable rate of decay. The pattern of demand is another important factor in the inventory system. Generally, demand is to be considered constant. Nevertheless, in some other production units dealing with food processing, mining, cement production and freight handling, demand depends on the selling price. The literature on stock models with selling price dependent has gathered further data.

Goel and Aggrawal (1980), Maiti, et al. (2009), Sena (2011), Srinivasa Rao, et al. (2007), Srinivasa Rao and Lakshmana Rao (2014), Teng, et al. (2005a) and Tripathy and Mishra (2010) studied inventory models with price-dependent demand sales. Essay and Srinivasa Rao (2012) have studied inventory models with selling price dependent demand and three parameter Weibull decay having stock dependent production.

Very little literature work has been done on randomly replenished stock models and Pareto decay with price dependent demand sales, which are very useful for obtaining optimal production schedules and ordering policies. Thus, this paper develops and analyzes an inventory model for the deterioration of items on the assumption that replenishment is random and follows an Exponential Distribution. The lifetime of the product is also assumed to be random and follows the distribution of Pareto and demand is assumed to be a linear function of the selling price. Assuming shortages are permitted and the instantaneous state of stock is completely backlogged. Use differential equations, the overall cost function and the rate of profit function are achieved. The optimal production schedule and optimal output quantity are obtained by optimizing the profit rate function. The sensitivity analysis is carried out by statistical comparison. The layout is generalized in the case of no shortages.

2.2 ASSUMPTIONS:

(i) The demand rate is depends on unit selling price which is $f(s) = a - b\,s$, $a>0$, $b<a$. (2.2.1)

(ii)The replenishment is finite and fits the density function of the Exponential distribution

$f(t) = \lambda e^{-\lambda t}, t > 0, \lambda > 0$

Therefore the instantaneous replenishment is $k(t) = \frac{f(t)}{1-F(t)} = \lambda,\ \lambda > 0$ (2.2.2)

(iii) Time of lead is zero

(iv) The length of the cycle T is fixed and known

(v) The shortages are permitted and completely backlogged

(vi) The deterioration unit has been lost

(vii) Instantaneous rate of deterioration is $h(t) = \frac{\alpha}{t}, t > 0.$ (2.2.3)

Notations of the model:

The use the following notations to the development of the model

Q: Order quantity in a single cycle

A: Ordering cost

C: Cost per unit

C_1: The cost of the inventory per unit of time

C_2: The cost of shortage per unit time

s: Price for selling per unit

F(s): Demand rate.

2.3 INVENTORY MODEL WITH SHORTAGES:

Consider the system of inventories where the inventory rate is at time t=0 is zero. Inventory levels increase over the time (0, t_1) due to additional demand replenishment and deterioration is met. When the inventory rate exceeds S, the replenishment ceases at time t_1. The stock is gradually decreasing due to demand and interval degradation (t_1, t_2). At t_2 the stock must generate null and back orders over the duration (t_2, t_3). At time t_3, after meeting the demand, the replenishment begins again and fulfills the backlog. The back orders are met during (t_3,T) and the stock level at the close of round T hits zero. The diagram schematic showing the instantaneous state of stock is shown in Figure 2.1

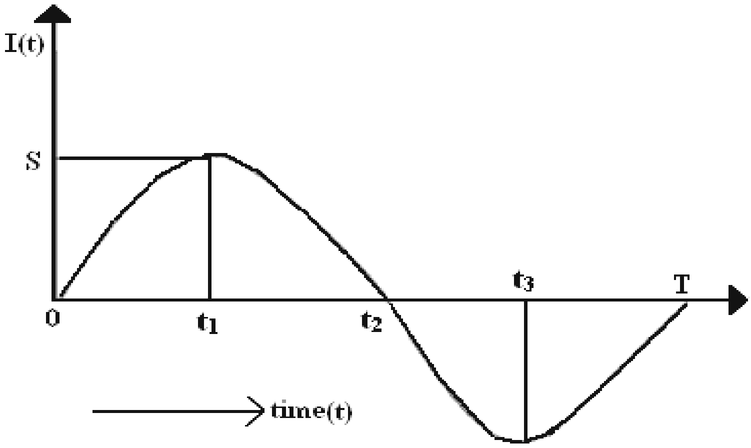


Fig 2.1: Diagram of schematics representing the inventory level.

Let I(t) be the system's inventory at ' t ' time ($0 \le t \le T$). Differential equations governing the instant I(t) status over the T phase duration.

$$\frac{d}{dt}I(t) + h(t)I(t) = \lambda - (a - bs), 0 \le t \le t_1 \quad (2.3.1)$$

$$\frac{d}{dt}I(t) + h(t)I(t) = -(a - bs), t_1 \le t \le t_2 \quad (2.3.2)$$

$$\frac{d}{dt}I(t) = -(a - bs), t_2 \le t \le t_3 \quad (2.3.3)$$

$$\frac{d}{dt}I(t) = \lambda - (a - bs), t_3 \le t \le T \quad (2.3.4)$$

where, h(t) is as mentioned in equation (2.2.3),under the initial conditions I(0) = 0, I(t_1) = S, I(t_2) = 0 and I(T) = 0.

Replace h (t) in the equation (2.3.3) in equations (2.3.1) and (2.3.2) and solving differential equations, the stock on hand shall be acquired as

$$I(t) = \frac{\lambda-(a-bs)}{\alpha+1}\left(\frac{t^{\alpha+1}-t_1^{\alpha+1}}{t^{\alpha}}\right) + S\left(\frac{t_1}{t}\right)^{\alpha}, 0 \leq t \leq t_1 \tag{2.3.5}$$

$$I(t) = \frac{-(a-bs)}{\alpha+1}\left(\frac{t^{\alpha+1}-t_1^{\alpha+1}}{t^{\alpha}}\right) + S\left(\frac{t_1}{t}\right)^{\alpha}, t_1 \leq t \leq t_2 \tag{2.3.6}$$

$$I(t) = (a-bs)(t_2 - t), t_2 \leq t \leq t_3 \tag{2.3.7}$$

$$I(t) = (\lambda - (a-bs))(t-T),\ t_3 \leq t \leq T \tag{2.3.8}$$

Loss of stock due to deterioration of the range (0, t)

$$L(t) = \int_0^t k(t)dt - \int_0^t (a-bs)dt - I(t)\,; 0 \leq t \leq t_2$$

This implies

$$L(t) = \begin{cases} \lambda t - (a-bs)t - \frac{\lambda-(a-bs)}{\alpha+1}\left(\frac{t^{\alpha+1}-t_1^{\alpha+1}}{t^{\alpha}}\right) - S\left(\frac{t_1}{t}\right)^{\alpha}; 0 \leq t \leq t_1 \\ \lambda t_1 - (a-bs)t_1 + \frac{(a-bs)}{\alpha+1}\left(\frac{t^{\alpha+1}-t_1^{\alpha+1}}{t^{\alpha}}\right) - S\left(\frac{t_1}{t}\right)^{\alpha};\ t_1 \leq t \leq t_2 \end{cases}$$

Loss of stock due to deterioration of the T-length cycle

$$L(T) = \lambda t_1 - (a-bs)t_2$$

The order quantity Q for the length cycle T is

$$Q = \int_0^{t_1} k(t)dt + \int_{t_3}^{T} k(t)dt = \lambda(\,t_1 + T - t_3) \tag{2.3.9}$$

From equation (2.3.5) and to use the initial conditions I (0) = 0, we get the value of 'S '

$$S = \frac{\lambda-(a-bs)}{\alpha+1}t_1 \tag{2.3.10}$$

From equation (2.3.6) and using the initial condition I (t_2) = 0, one can get

$$t_2 = \left(t_1^{\alpha+1} + \frac{St_1^{\alpha}(\alpha+1)}{(a-bs)}\right) \tag{2.3.11}$$

On simplification

$$t_2 = \left(\frac{\lambda t_1^{\alpha+1}}{a-bs}\right)^{\frac{1}{\alpha+1}} \tag{2.3.12}$$

Let $K(t_1, t_2, t_3, s)$ be the total cost per time unit. Since the total cost is the amount of the cost collection, the cost of the items, the cost of keeping the stock, the total cost per unit time is.

$$K(t_1, t_2, t_3, s) = \frac{A}{T} + \frac{CQ}{T} + \frac{h}{T}\left(\int_0^{t_1} I(t)dt + \int_{t_1}^{t_2} I(t)dt\right) + \frac{\pi}{T}\left(\int_{t_2}^{t_3} -I(t)dt + \int_{t_3}^{T} -I(t)dt\right) \tag{2.3.13}$$

Substitute the values of I(t) and Q in the equation (2.3.13) one could obtain $K(t_1, t_2, t_3, s)$ as

$$K(t_1, t_2, t_3, s) = \frac{A}{T} + \frac{C}{T}\lambda(\,t_1 + T - t_3) + \frac{C_1}{T}\left[\int_0^{t_1}\left(\frac{\lambda-(a-bs)}{\alpha+1}\left(\frac{t^{\alpha+1}-t_1^{\alpha+1}}{t^{\alpha}}\right) + S\left(\frac{t_1}{t}\right)^{\alpha}\right)dt\right.$$

$$+\int_{t_1}^{t_2}\left(\frac{-(a-bs)}{\alpha+1}\left(\frac{t^{\alpha+1}-t_1^{\alpha+1}}{t^{\alpha}}\right)+S\left(\frac{t_1}{t}\right)^{\alpha}\right)dt\Bigg]+\frac{C_2}{T}\Bigg[\int_{t_2}^{t_3}\big((a-bs)(t-t_2)\big)dt+\int_{t_3}^{T}\big((\lambda-(a-bs))(t-T)\big)dt\Bigg] \qquad (2.3.14)$$

On simplification

$$K(t_1,t_2,t_3,s)=\frac{A}{T}+\frac{C}{T}\lambda(\,t_1+T-t_3)+\frac{C_1}{T}\Bigg[\frac{\lambda}{\alpha+1}\left(\frac{t_1^2}{2}-\frac{t_1^2}{1-\alpha}\right)-\frac{(a-bs)}{\alpha+1}\left(\frac{t_2^2}{2}-\frac{t_1^{\alpha+1}t_2^{-\alpha+1}}{1-\alpha}\right)+\frac{St_2^{-\alpha+1}t_1^{\alpha}}{1-\alpha}\Bigg]+\frac{C_2}{T}\Bigg[(a-bs)\left(\frac{t_2^2}{2}-t_2t_3-\frac{T^2}{2}+Tt_3\right)+\lambda\left(\frac{T^2}{2}-Tt_3+\frac{t_3^2}{2}\right)\Bigg] \qquad (2.3.15)$$

Let $P(t_1,t_2,t_3,s)$ be the function of profit rate. Since the rate of profit is the total income per unit minus the total cost per unit time, we have.

$$P(t_1,t_2,t_3,s)\;=\mathrm{s}\,(a-bs)-K(t_1,t_2,t_3,s) \qquad (2.3.16)$$

Substituting the value of $K(t_1,t_2,t_3,s)$ in equation (2.3.16), one can get the profit rate function as

$$P(t_1,t_2,t_3,s)=\;\mathrm{s}\,(a-bs)-\Bigg[\frac{A}{T}+\frac{C}{T}\lambda(\,t_1+T-t_3)+\frac{C_1}{T}\Bigg[\frac{\lambda}{\alpha+1}\left(\frac{t_1^2}{2}-\frac{t_1^2}{1-\alpha}\right)-\frac{(a-bs)}{\alpha+1}\left(\frac{t_2^2}{2}-\frac{t_1^{\alpha+1}t_2^{-\alpha+1}}{1-\alpha}\right)+\frac{St_2^{-\alpha+1}t_1^{\alpha}}{1-\alpha}\Bigg]+\frac{C_2}{T}\Bigg[(a-bs)\left(\frac{t_2^2}{2}-t_2t_3-\frac{T^2}{2}+Tt_3\right)+\lambda\left(\frac{T^2}{2}-Tt_3+\frac{t_3^2}{2}\right)\Bigg]\Bigg] \qquad (2.3.17)$$

Substituting equations (2.3.10) and (2.3.12) in equation (2.3.17), one can get the profit rate function in terms of ‘t_1’, ‘t_3’ and ’s’ as

$$P(t_1,t_3,s)=\;\mathrm{s}\,(a-bs)-\frac{A}{T}-\frac{C}{T}\lambda(\,t_1+T-t_3)-\frac{C_1}{T}\Bigg[\frac{\lambda}{\alpha+1}\left(\frac{t_1^2}{2}-\frac{t_1^2}{1-\alpha}\right)-\frac{(a-bs)}{\alpha+1}\left(\frac{1}{2}\left(\frac{\lambda t_1^{\alpha+1}}{a-bs}\right)^{\frac{2}{\alpha+1}}-\frac{t_1^{\alpha+1}}{1-\alpha}\left(\frac{\lambda t_1^{\alpha+1}}{a-bs}\right)^{\frac{-\alpha+1}{\alpha+1}}\right)+\frac{t_1^{\alpha+1}}{1-\alpha}\left(\frac{\lambda-(a-bs)}{\alpha+1}\right)\left(\frac{\lambda t_1^{\alpha+1}}{a-bs}\right)^{\frac{-\alpha+1}{\alpha+1}}\Bigg]$$

$$-\frac{C_2}{T}\Bigg[(a-bs)\left(\frac{t_2^2}{2}-t_2t_3-\frac{T^2}{2}+Tt_3\right)+\lambda\left(\frac{T^2}{2}-Tt_3+\frac{t_3^2}{2}\right)\Bigg]$$

On simplification

$$P(t_1,t_3,s)=\;\mathrm{s}\,(a-bs)-\frac{A}{T}-\frac{C}{T}\lambda(\,t_1+T-t_3)-\frac{C_1}{T}\Bigg[\frac{\lambda}{\alpha+1}\left(\frac{t_1^2}{2}-\frac{t_1^2}{1-\alpha}+\frac{t_1^2}{1-\alpha}\left(\frac{\lambda}{a-bs}\right)^{\frac{-\alpha+1}{\alpha+1}}\right)-\frac{(a-bs)}{\alpha+1}\left(\frac{t_1^2}{2}\left(\frac{\lambda}{a-bs}\right)^{\frac{2}{\alpha+1}}\right)\Bigg]-\frac{C_2}{T}\Bigg[(a-bs)\left(\frac{t_2^2}{2}\left(\frac{\lambda}{a-bs}\right)-t_1\left(\frac{\lambda}{a-bs}\right)^{\frac{1}{\alpha+1}}t_3-\frac{T^2}{2}+Tt_3\right)+\lambda\left(\frac{T^2}{2}-Tt_3+\frac{t_3^2}{2}\right)\Bigg] \qquad (2.3.18)$$

2.4 OPTIMAL POLICIES AND PRICING OF THE MODEL:

We get the optimum stock process policies under review in this section. We get the first order partial derivatives of $P(t_1, t_3, s)$ given in equation (2.3.18) with respect to t_1 and s and equal to zero to find optimal t_1 and s values. The maximization criterion of $P(t_1, t_3, s)$ is

$$D = \begin{bmatrix} \frac{\partial^2 P(t_1,t_3,s)}{\partial t_1^2} & \frac{\partial^2 P(t_1,t_3,s)}{\partial t_1, \partial t_3} & \frac{\partial^2 P(t_1,t_3,s)}{\partial t_1, \partial s} \\ \frac{\partial^2 P(t_1,t_3,s)}{\partial t_1, \partial t_3} & \frac{\partial^2 P(t_1,t_3,s)}{\partial t_3^2} & \frac{\partial^2 P(t_1,t_3,s)}{\partial t_3, \partial s} \\ \frac{\partial^2 P(t_1,t_3,s)}{\partial t_1, \partial s} & \frac{\partial^2 P(t_1,t_3,s)}{\partial t_3, \partial s} & \frac{\partial^2 P(t_1,t_3,s)}{\partial s^2} \end{bmatrix} < 0$$

Differentiating $P(t_1, t_3, s)$ given in equation (2.3.18) with respect to t_1 and zero, you can get

$$\frac{C\lambda}{T} + \frac{C_1}{T}\left[\frac{\lambda}{\alpha+1}\left(t_1 - \frac{2t_1}{1-\alpha} + \frac{2t_1}{1-\alpha}\left(\frac{\lambda}{a-bs}\right)^{\frac{-\alpha+1}{\alpha+1}}\right) - \frac{(a-bs)}{\alpha+1}\left(t_1\left(\frac{\lambda}{a-bs}\right)^{\frac{2}{\alpha+1}}\right)\right] \qquad + \frac{C_2}{T}\left[(a-bs)\, t_1\left(\frac{\lambda}{a-bs}\right) - \left(\frac{\lambda}{a-bs}\right)^{\frac{1}{\alpha+1}} t_3\right] = 0 \tag{2.4.1}$$

Differentiating $P(t_1, t_3, s)$ given in equation (2.3.18) with respect to t_3 and zero, you can get

$$\frac{C\lambda}{T} - \frac{C_2}{T}\left[\left(T - \left(\frac{\lambda}{a-bs}\right)^{\frac{1}{\alpha+1}} t_1\right)(a-bs) + \lambda(t_3 - T)\right] = 0 \tag{2.4.2}$$

Differentiating $P(t_1, t_3, s)$ given in equation (2.3.18) with respect to s and zero, you can get

$$(a-bs) - \frac{C_1}{T}\left[\left[\frac{\lambda}{(\alpha+1\,)(1-\alpha)}\frac{t_1^2}{}\lambda^{\frac{1-\alpha}{1+\alpha}}\left(\frac{\alpha-1}{\alpha+1}\right)(a-bs)^{\frac{-2}{\alpha+1}} - \frac{\lambda^{\frac{2}{1+\alpha}}}{2(\alpha+1)}t_1^2\left(\frac{\alpha-1}{\alpha+1}\right)(a-bs)^{\frac{-2}{\alpha+1}}\right]\right.$$
$$\left. - \frac{C_2}{T}\left[\frac{t_1^2}{2}\lambda^{\frac{2}{1+\alpha}}\left(\frac{\alpha-1}{\alpha+1}\right)(a-bs)^{\frac{-2}{\alpha+1}} - t_1 t_3 \lambda^{\frac{1}{1+\alpha}}\frac{\alpha}{\alpha+1}(a-bs)^{\frac{-1}{\alpha+1}} + b\left(\frac{T^2}{2} + Tt_3\right)\right] \quad = 0 \tag{2.4.3}$$

Solving the equations (2.4.1), (2.4.2) and (2.4.3) at the same time, we obtain the optimum time at which replenishment t_1^* of t_1 is stopped and the optimal time t_3^* of t_3 at which replenishment is restarted after back-order accumulation and optimum selling price s^* of s is obtained.

The optimum quantity order Q * of Q in the T-length cycle is obtained by substituting the optimum values of t_1^*, t_3^* in the equation. (2.3.9) as

$$Q^* \ = \lambda(t_1^* + T - t_3^*) \tag{2.4.4}$$

2.5 NUMERICAL ILLUSTRATION:

We are addressing this section the model's solution by means of numerical illustration by inventory system replenishment (production) uptime, replenishment (production) downtime, optimum order quantity and total profit. It was assumed here that the commodity deteriorated and that shortages were permitted and completely backlogged. In order to demonstrate the model's solution procedure, the other parameter values and the model-related costs are as follows:

A = 2000, 2100, 2200, 2300; C = 8.5, 8.925, 9.35, 9.775,
C_1 = 20, 21, 22, 23; C_2= 0.05, 0.0525, 0.055, 0.0575,
α = 0.5, 0.525, 0.55, 0.575; λ= 0.9, 0.945, 0.99, 1.44,
a= 20, 21, 22, 23; b = 0.5, 0.525, 0.55, 0.575; T = 12 months.

Substitute optimal order quantity Q*, replenishment(production) uptime, replenishment (production)down time, optimal selling price, total profit are calculated and presented in Table 2.1.

Table 2.1 show that the deterioration and replenishment parameters have an enormous impact on optimum replenishment times, order size, selling price and total profit.

If cost of ordering 'A' increased from 2000 to 2300, then optimum downtime replenishment t_1^* increased from 4.68 to 6.3.8, optimum uptime replenishment t_3^* increased from 5.291 to 7.157, the optimal selling price s^* increases from 17.341 to 19.632.31, optimal quantity of order Q^* decreases from 10.059 to 9.481, the total profit P^* decreases from 40.252 to 34.473. The cost parameter C increased from 8 to 9.775, optimum down replenishment time increased from 4.468 to 4.915, optimum uptime replenishment decreases from 5.291 to 5.225, the optimal selling price decreases from 17.341 to 17.365, optimal quantity of order Q^* increased from 10.059 to 10.521 and the total profit increases from 40.252 to 42;.5.

As the cost of keeping stock 'C_1' increased from 20 to 23, then optimum downtime replenishment t_1^* decreases from 4.468 to 4.055, the uptime replenishment t_3^* increased from 5.291 to 5.443, the selling price increased from 17.341 to 17.347 optimal quantity of order decreases from 10.059 to 9.550 and the total profit decreases from 40.252 to 39.362. As the shortage cost 'C_2' increased from 0.05 to 0.0575, optimum downtime replenishment decreases from 4.468 to 4.351, optimum uptime replenishment decreases from 5.291 to 4.798, the optimal selling price decreases from 17.341 to 17.340, optimal quantity of order increased from 10.059 to 10.398 and total profit decreases from 40.252 to 39.486.

Table 2.1

Optimum t_1 *, t_3 *, s *, Q * and P * values for various parameter values

A	C	C_1	C_2	T	λ	α	a	b	t_1	t_3	s	Q	P
2000	**8.5**	**20**	**0.05**	**12**	**0.5**	**0.9**	**20**	**0.5**	4.468	5.291	17.341	10.059	40.252
2100				**12**					4.963	6.166	18.157	9.718	37.377
2200				**12**					5.668	6.638	18.907	9.626	35.912
2300				**12**					6.358	7.157	19.632	9.481	34.473
	8.925			**12**					4.632	5.272	17.346	10.224	41.076
	9.350			**12**					4.780	5.250	17.354	10.377	41.824
	9.775			**12**					4.915	5.225	17.365	10.521	42.500
		21		**12**					4.334	5.321	17.341	9.912	40.129
		22		**12**					4.195	5.373	17.343	9.740	39.916
		23		**12**					4.055	5.443	17.347	9.550	39.632
			0.0525	**12**					4.436	5.133	17.340	10.172	40.033
			0.0550	**12**					4.396	4.966	17.340	10.287	39.773
			0.0575	**12**					4.351	4.798	17.341	10.398	39.486
				12	**0.525**				4.436	5.314	17.343	10.009	40.164
				12	**0.550**				4.404	5.337	17.345	9.960	40.072
				12	**0.575**				4.372	5.361	17.347	9.910	39.979
				12		**0.945**			4.397	5.426	17.372	10.368	40.116
				12		**0.990**			4.336	5.526	17.401	10.702	40.006
				12		**1.035**			4.335	5.527	17.402	10.710	40.004
				12			**21**		4.127	5.351	17.205	9.698	39.366
				12			**22**		3.751	6.784	23.001	8.071	38.710
				12			**23**		3.741	6.642	22.924	8.189	39.077
				12				**0.525**	5.142	5.682	17.869	10.315	38.337
				12				**0.550**	6.027	6.406	18.371	10.459	37.740
				12				**0.575**	6.842	7.201	18.804	10.477	36.296

As replenishment parameter 'λ' increased from 0.5 to 0.575 units, optimum down time replenishment decreases from 4.468 to 4.372, optimum uptime replenishment increased from 5.291 to 5.361, the optimal selling price increases from 17.341 to 17.347, optimal quantity of order Q^* decreases from 10.059 to 9.91 and the total profit decreases from 40.252 to 39.979. As deteriorating parameter 'α' increased from 0.9 to 1.035, optimum down time replenishment increased from 4.252 to 4.335, optimum uptime replenishment increased from 5.291 to 5.527, the optimal selling price increased from 17.341 to 17.402, optimal quantity of order Q^* increases from 10.059 to 10.710, the total profit decreases from 40.252 to 40.004.

As demand parameter 'a' increased from 20 to 23 units, the optimum value of t_1^* decreases from 4.468 to 3.741, the optimum value of t_3^* increased from 5.291 to 6.642, the optimum value of s^* increased from 17.341 to 22.924, the optimum value of Q^* decreases from 10.059 to 8.189, the total profit P^* decreases from 40.252 to 39.077. Another demand parameter 'b' increased from 0.5 to 0.575, optimum down time replenishment increased from 4.486 to 6.842, optimum uptime replenishment increased from 5.291 to 7.201, the optimal selling price increases from 17.341 to 18.804, optimal quantity of order increased from 10.059 to 10.477, the total profit decreases from 40.252 to 36.296.

2.6 SENSITIVITY ANALYSIS OF THE MODEL:

The sensitivity analysis is carried out to analyze the effect on optimal policies of changes in process parameters and costs by varying the parameter at a time for the model being evaluated (-15%, -10%,-5%, 0%, 5%, 10%, 15%). The findings are shown in Table 2.2. Figure 2.2 shows the relationship between the optimum values and the parameters.

It is found that the costs affect the optimal order schedules of quantities and replenishment significantly. As the cost of ordering A decreases, optimum downtime replenishment t_1^*, the optimum uptime replenishment t_3^*, optimal selling price s^* are decreasing and optimum quantity of order Q^* and total profit P^* are increases. As ordering cost A increased, the optimum down time replenishment t_1^*, optimum uptime replenishment t_3^*, optimum selling price s^* are increases and optimum quantity of order Q^* and total profit $P^*$are decreasing. As t cost per unit C decreases, optimum uptime replenishment t_3^* and optimal selling price s^* are increases and optimum down time replenishment t_1^*, optimum quantity of order $Q^*$and total profit P^* are decreases. As cost per unit C increased, optimum uptime replenishment t_3^* and optimal selling price s^* are decreases and optimal down time replenishment t_1^*, optimum quantity of ordering Q^* and total profit P^* are increases.

The optimum values of t_1 *, t_3 *, s * Q*and P * are increasing as the holding cost ' C_1 ' decreases. The optimum values of t_1 *, t_3 *, s * Q*and P * are decreasing as the holding cost ' C_1 ' increases. As shortage cost ‘C_2’ decreases, optimum uptime replenishment t_3^*, the optimum selling price s^* and the total profit $P^*$are increases and optimal down time replenishment t_1^* and optimum quantity of order Q^* are decreasing. As shortage cost ‘C_2’ increases, the optimum uptime replenishment t_3^*, the optimum selling price s^* and the total profit $P^*$are decreasing and optimum down time replenishment t_1^* and optimum quantity of order Q^* are increases.

As replenishment parameter ‘λ’ decreases, the optimum values of t_1^*, Q^* and P^* are increases, the optimum value of t_3^*, s^* are decreases. As replenishment parameter ‘λ’ increases, the optimum values of t_1^*, Q^* and P^* are decreases and the optimum value of t_3^*, s^* are increases. The deteriorating parameter ‘α’ decreases, the optimum values of t_1^*, t_3^* $s^*$and P^* are increases and optimum ordering quantity Q^* is decrease. The deteriorating parameter ‘α’ increases, the optimum values of t_1^*, t_3^* $s^*$and P^* are decreases and optimum quantity of order Q^* is increase.

Table 2.2
System sensitivity analysis-with shortages

	Optimum	**Change of parameters**						
Parameters	**Policies**	**-15%**	**-10%**	**-5%**	**0%**	**5%**	**10%**	**15%**
A	t_1^*	1.851	1.85	2.652	4.468	4.963	5.668	6.358
	t_3^*	3.991	3.996	4.94	5.291	6.166	6.638	7.157
	s^*	16.610	16.611	17.308	17.341	18.157	18.907	19.632
	Q^*	10.468	10.369	10.241	10.059	9.718	9.626	9.081
	P^*	42.749	42.649	41.053	40.252	37.377	35.912	34.473
C	t_1^*	4.078	4.077	4.284	4.468	4.632	4.78	4.915
	t_3^*	5.321	5.321	5.307	5.291	5.272	5.25	5.225
	s^*	17.341	17.341	17.339	17.341	17.346	17.354	17.365
	Q^*	9.681	9.68	9.88	10.059	10.224	10.377	10.521
	P^*	38.321	38.316	39.335	40.252	41.076	41.824	42.5
C_1	t_1^*	4.789	4.704	4.593	4.468	4.334	4.195	4.055
	t_3^*	5.424	5.331	5.292	5.291	5.321	5.373	5.443
	s^*	17.368	17.353	17.345	17.341	17.341	17.343	17.347
	Q^*	10.228	10.235	10.172	10.059	9.912	9.74	9.55
	P^*	40.442	40.306	40.257	40.252	40.129	39.916	39.632
C_2	t_1^*	4.532	4.515	4.494	4.468	4.436	4.396	4.351
	t_3^*	5.73	5.587	5.441	5.291	5.133	4.966	4.798
	s^*	17.349	17.346	17.343	17.341	17.34	17.34	17.341
	Q^*	9.721	9.835	9.947	10.059	10.172	10.287	10.398
	P^*	40.743	40.6	40.437	40.252	40.033	39.773	39.486
α	t_1^*	4.566	4.533	4.5	4.468	4.436	4.404	4.372
	t_3^*	5.228	5.248	5.269	5.291	5.314	5.337	5.361
	s^*	17.336	17.338	17.339	17.341	17.343	17.345	17.347
	Q^*	10.204	10.156	10.108	10.059	10.009	9.96	9.91
	P^*	40.495	40.418	40.337	40.252	40.164	40.072	39.979
λ	t_1^*	4.626	4.628	4.545	4.468	4.397	4.336	4.335
	t_3^*	4.863	4.857	5.104	5.291	5.426	5.526	5.527
	s^*	17.28	17.28	17.31	17.341	17.372	17.401	17.402
	Q^*	9.54	9.535	9.782	10.059	10.368	10.702	10.71
	P^*	40.532	40.536	40.395	40.252	40.116	40.006	40.004
a	t_1^*	8.041	7.888	6.102	4.468	4.127	3.751	3.745
	t_3^*	6.295	6.24	5.837	5.291	5.151	4.784	4.714
	s^*	20.495	20.441	18.977	17.341	17.205	16.001	15.963
	Q^*	10.572	10.484	10.138	10.059	9.698	8.071	8.128
	P^*	41.07	40.653	40.513	40.252	39.366	38.71	38.69
b	t_1^*	3.8	4.097	4.29	4.468	5.142	6.027	6.842
	t_3^*	6.586	5.87	5.569	5.291	5.682	6.406	7.201
	s^*	17.125	17.255	17.347	17.341	17.869	18.371	18.804
	Q^*	8.293	9.204	9.649	10.059	10.315	10.459	10.477
	P^*	40.786	40.478	40.183	40.252	38.337	37.74	36.296

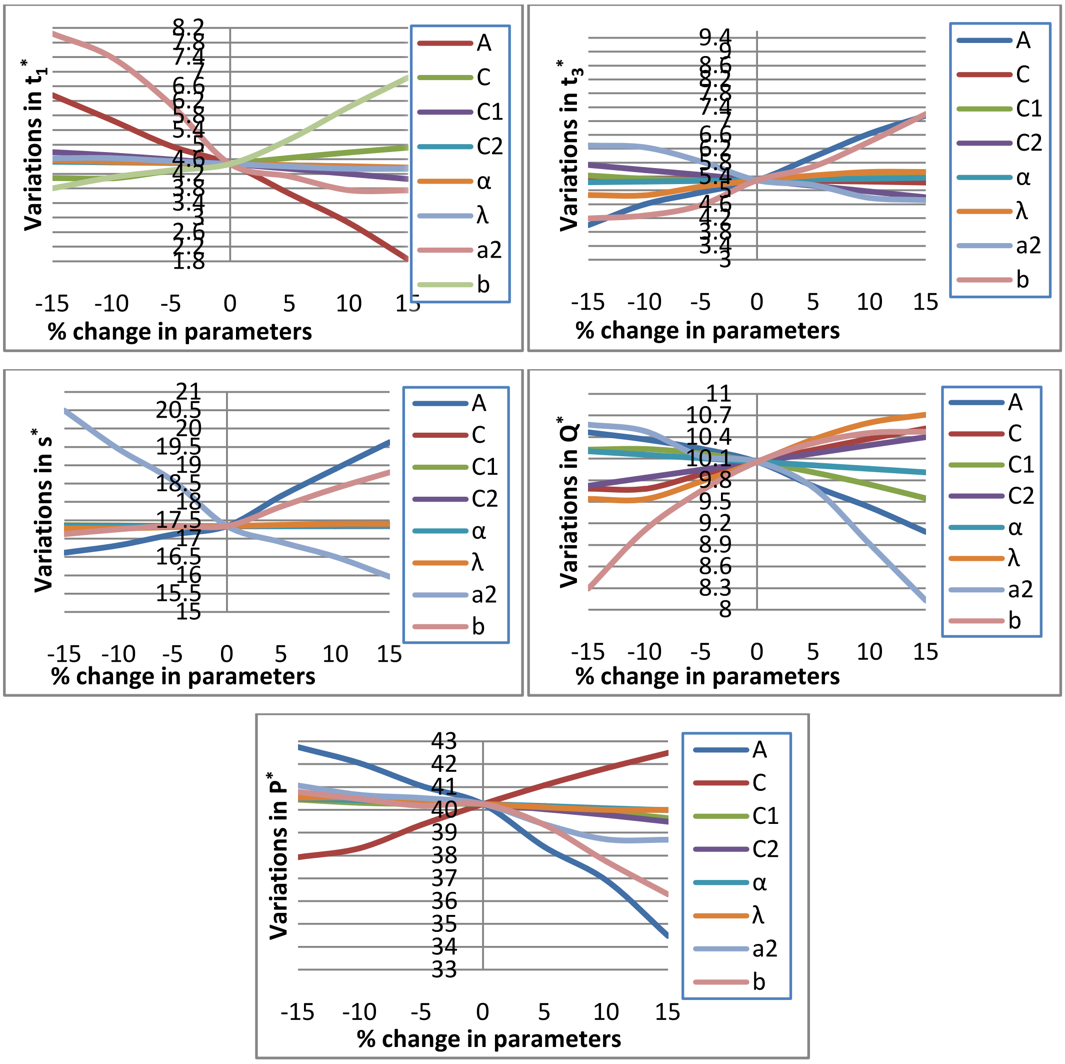


Fig 2.2: Relationship between parameters and optimum shortage values

The demand parameter ‘a’ decreases, the optimum values of t_1^*, t_3^*, s^* $Q^*$and P^* are increases. The demand parameter ‘a’ increases, the optimum values of t_1^*, t_3^*, s^* $Q^*$and P^* are decreases. As another demand parameter ‘b’ decreases, the optimum values of t_3^*, P^* are increases and the optimum values of t_1^*, s^* and $Q^*$are decreases. As another demand parameter ‘b’ increases, the optimum values of t_3^*, P^* are decreases and the optimum values of t_1^*, s^* and $Q^*$are increases.

2.7 INVENTORY MODEL WITHOUT SHORTAGES:

In this section, the stock model is built and evaluated to deteriorate products without shortages. Here, shortages are considered not to be permitted and the inventory rate at time t= 0 is zero. During the period (0, t_1), the stock level increases due to excessive demand replenishment satisfaction and deteriorating. When the stock level reaches S, the replenishment stops at time t_1. The inventory is gradually decreasing due to demand and interval deterioration (t_1, T). The stock is zero at the time T. The diagram of schematic showing the instantaneous state of stock is shown in Figure 2.3

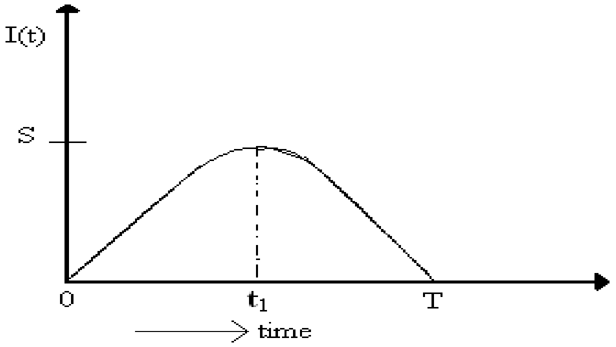


Fig 2.3: Diagram of schematics representing the inventory level.

Let I(t) be the inventory level of the system at ' t ' time (0≤t≤T). Differential equations that govern the instant state of I(t) over the duration of the T phase.

$$\frac{d}{dt}I(t) + h(t)I(t) = \lambda - (a - bs), 0 \le t \le t_1 \tag{2.7.1}$$

$$\frac{d}{dt}I(t) + h(t)I(t) = -(a - bs), t_1 \le t \le t_2 \tag{2.7.2}$$

where, h(t) is as mentioned in equation (2.2.3),under the initial conditions I(0) = 0, I(t_1) = S, I(t_2) = 0 and I(T) = 0.

Replace h(t) from equation (2.2.3) in equation (2.7.1) and (2.7.2) and solving differential equations, stock on hand shall be acquired as

$$I(t) = \frac{\lambda-(a-bs)}{\alpha+1}\left(\frac{t^{\alpha+1}-t_1^{\alpha+1}}{t^{\alpha}}\right) + S\left(\frac{t_1}{t}\right)^{\alpha}, 0 \le t \le t_1 \tag{2.7.3}$$

$$I(t) = \frac{-(a-bs)}{\alpha+1}\left(\frac{t^{\alpha+1}-t_1^{\alpha+1}}{t^{\alpha}}\right) + S\left(\frac{t_1}{t}\right)^{\alpha}, t_1 \le t \le t_2 \tag{2.7.4}$$

Loss of sotck due to deterioration of the range (0, t)

$$L(t) = \int_0^t k(t)dt - \int_0^t (a - bs)dt - I(t), 0 \le t \le T$$

This implies

$$L(t) = \begin{cases} \lambda t - (a - bs)t - \frac{\lambda-(a-bs)}{\alpha+1}\left(\frac{t^{\alpha+1}-t_1^{\alpha+1}}{t^{\alpha}}\right) - S\left(\frac{t_1}{t}\right)^{\alpha}; 0 \le t \le t_1 \\ \lambda t_1 - (a - bs)t_1 + \frac{(a-bs)}{\alpha+1}\left(\frac{t^{\alpha+1}-t_1^{\alpha+1}}{t^{\alpha}}\right) - S\left(\frac{t_1}{t}\right)^{\alpha};\ t_1 \le t \le T \end{cases}$$

The order quantity Q for the length cycle T is

$$Q = \int_0^{t_1} k(t)dt = \lambda t_1 \tag{2.7.5}$$

From equation (2.7.3) and to use the initial conditions I (0) = 0, we get the value of 'S '

$$S = \frac{\lambda-(a-bs)}{\alpha+1}t_1 \tag{2.7.6}$$

Let the total cost per unit time be K(t_1 ,s). Since the total cost is the sum of the cost of set-up, the cost of units, the cost of holding inventory. Therefore the total cost is

$$K(t_1, s) = \frac{A}{T} + \frac{CQ}{T} + \frac{C_1}{T}\left(\int_0^{t_1} I(t)dt + \int_{t_1}^{T} I(t)dt\right) \tag{2.7.7}$$

Replacement of the value of I (t) and Q in the equation (2.7.3), (2.7.4) and (2.7.5) in equation (2.7.7), we obtain $K(t_1, s)$ as

$$K(t_1, s) = \frac{A}{T}\lambda t_1 + \frac{C_1}{T}\Bigg\{\int_0^{t_1}\Bigg[\frac{\lambda - (a - bs)}{\alpha + 1}\left(\frac{t^{\alpha+1} - t_1^{\alpha+1}}{t^{\alpha}}\right) + S\left(\frac{t_1}{t}\right)^{\alpha} dt\Bigg]$$

$$+ \int_{t_1}^{T}\Bigg[-\frac{(a - bs)}{\alpha + 1}\left(\frac{t^{\alpha+1} - t_1^{\alpha+1}}{t^{\alpha}}\right) + S\left(\frac{t_1}{t}\right)^{\alpha} dt\Bigg]\Bigg\}$$

On simplification we get

$$K(t_1, s) = \frac{A}{T} + \frac{C}{T}\lambda t_1 + \frac{C_1}{T}\left[\frac{\lambda}{\alpha+1}\left(\frac{t_1^2}{2} - \frac{t_1^2}{1-\alpha}\right) - \frac{(a-bs)}{\alpha+1}\left(\frac{T^2}{2} - \frac{t_1^{\alpha+1}T^{1-\alpha}}{1-\alpha}\right) + \frac{S}{1-\alpha}t_1^{\alpha}T^{1-\alpha}\right] \tag{2.7.8}$$

Let P(t_1 ,s) be the function of the profit rate. Since the function of the profit rate is the total revenue per unit minus the total cost per unit time, we have

$$P(t_1, s) = \text{s}\,(a - bs) - K(t_1, s) \tag{2.7.9}$$

Substituting $K(t_1, s)$ value given in equation (2.7.8) in equation (2.7.9), one can get the rate of profit function as

$$P(t_1, s) = \text{s}\,(a - bs) - \frac{A}{T} - \frac{C}{T}\lambda t_1 - \frac{C_1}{T}\left[\frac{\lambda}{\alpha+1}\left(\frac{t_1^2}{2} - \frac{t_1^2}{1-\alpha}\right) - \frac{(a-bs)}{\alpha+1}\left(\frac{T^2}{2} - \frac{t_1^{\alpha+1}T^{1-\alpha}}{1-\alpha}\right) + \frac{S}{1-\alpha}t_1^{\alpha}T^{1-\alpha}\right] \tag{2.7.10}$$

Substituting ‘S’ value in equation (2.7.6) in equation (2.7.10), we get the profit rate function as

$$P(t_1, s) = \text{s}\,(a - bs) - \frac{A}{T} - \frac{C}{T}\lambda t_1 - \frac{C_1}{T}\Bigg[\frac{\lambda}{\alpha+1}\left(\frac{t_1^2}{2} - \frac{t_1^2}{1-\alpha}\right) - \frac{(a-bs)}{\alpha+1}\left(\frac{T^2}{2} - \frac{t_1^{\alpha+1}T^{1-\alpha}}{1-\alpha}\right)$$

$$+ \frac{\lambda - (a - bs)}{(\alpha + 1)(1 - \alpha)}t_1^{\alpha+1}T^{1-\alpha}\Bigg]$$

On simplification we get

$$P(t_1, s) = \text{s}\,(a - bs) - \frac{A}{T} - \frac{C}{T}\lambda t_1 - \frac{C_1}{T}\left[\frac{\lambda}{\alpha+1}\left(\frac{t_1^2}{2} - \frac{t_1^2}{1-\alpha} + \frac{t_1^{\alpha+1}T^{1-\alpha}}{1-\alpha}\right) - \frac{(a-bs)}{2(\alpha+1)}T^2\right] \tag{2.7.11}$$

2.8. OPTIMAL PRICING AND POLICIES OF THE MODEL:

This section we get the optimal inventory system policies that are being studied. We equate the first order partial derivatives of P(t_1,s) with respect to t_1 in order to find the optimal values of t_1 and s and equate them to zero. The maximization requirement for P(t_1 ,s) is

$$D = \begin{vmatrix} \frac{\partial^2 P(t_1,s)}{\partial t_1^2} & \frac{\partial^2 P(t_1,s)}{\partial t_1 \partial s} \\ \frac{\partial^2 P(t_1,s)}{\partial t_1 \partial s} & \frac{\partial^2 P(t_1,s)}{\partial s^2} \end{vmatrix} < 0$$

Differentiate $P(t_1, s)$ with respect to t_1 and equating to zero, you can get

$$-\frac{C}{T}\lambda - \frac{C_1}{T}\left[\frac{\lambda}{\alpha+1}\left(t_1 - \frac{2t_1}{1-\alpha} + \frac{1+\alpha}{1-\alpha}t_1^{\alpha+1}T^{1-\alpha}\right)\right] = 0 \quad (2.8.1)$$

Differentiate $P(t_1, s)$ with respect to ‘s’ and equating to zero, you can get

$$-2bs - \frac{C_1}{T}\left(\frac{bT^2}{2(\alpha+1)}\right) = 0 \quad (2.8.2)$$

Through simultaneously resolving the equations (2.8.1) and (2.8.2), we obtain the optimum time at which the replenishment of t_1* of t_1 is to be stopped and the optimum unit selling price of s*. The optimum ordering Q* of Q in the length T cycle is obtained by replacing the optimal values of t_1 in (2.7.5) as.

$$Q^* = \lambda t_1 \quad (2.8.3)$$

2.9. NUMERICAL ILLUSTRATION:

In this section we analyze the model's solution method through a numerical example by obtaining the replenishment (production) time, optimum order size, and total profit. There, it believed that the product would deteriorate in nature and that shortages would not be tolerated and thoroughly reported away. In order to demonstrate the model's solution process, the values of the model's parameters and costs are:

A = 175, 183.75, 192.5, 203; C = 10, 10.5, 11, 11.5,
C_1 = 10, 9.5, 9, 8.5; λ= 5, 5.25, 5.5, 5.75, α= 0.5, 0.525, 0.55, 0.575
a= 25, 26.25, 27.5, 28.75; b = 0.5, 0.525, 0.55, 0.575; T = 12 months.

Replacing these values is calculated and presented in Table 3 the optimal order quantity Q^*, replenishment time, optimal selling price and optimal profit per unit time.

Table 2.3 shows that the parameters of deterioration and replenishment have a tremendous influence on the model's optimal values.

Table 2.3

Optimum t_1 *, s *, Q * and P * values for various parameter values

A	**C**	**C_1**	**T**	**A**	**λ**	**a**	**b**	**t_1**	**s**	**Q**	**P**
175.00	**10**	**10**	**12**	**0.5**	**5**	**25**	**0.5**	4.061	29.902	20.306	422.25
183.75			**12**					4.06	29.887	20.302	421.975
192.50			**12**					4.06	29.873	20.298	421.7
201.25			**12**					4.059	29.859	20.294	421.425
	10.5		**12**					4.056	29.897	20.281	422.163
	11.0		**12**					4.051	29.892	20.256	422.076
	11.5		**12**					4.046	29.887	20.231	421.988
		9.5	**12**					4.011	30.195	20.057	413.505
		9.0	**12**					4.001	30.397	20.005	402.298
		8.5	**12**					3.99	30.616	19.95	391.012
			12	**0.525**				4.086	30.061	20.429	420.922
			12	**0.550**				4.151	30.115	20.753	417.308
			12	**0.575**				4.495	28.775	22.474	413.804
			12		**5.25**			3.967	29.869	20.828	422.154
			12		**5.50**			3.874	29.864	21.309	422.067
			12		**5.75**			3.326	35.664	19.127	319.985
			12			**26.25**		4.19	31.934	20.948	443.006
			12			**27.50**		4.361	33.887	21.806	460.449
			12			**28.75**		5.946	31.135	29.728	365.349
			12				**0.525**	4.384	27.578	21.919	388.402
			12				**0.550**	5.654	21.568	28.27	288.672
			12				**0.575**	5.176	22.487	25.878	314.179

When ordering cost A increased from 175 to 201.25, the optimum replenishment time t_1^* decreases from 4.61 to 4.046, the optimum selling price s^* decreases from 29.902 to 29.887, the optimum quantity order Q^* decreases from 20.306 to 20.294, the total profit P^* decreases from 422.25 to 421.425. As cost per unit ‘C’ increased from 10 to 11.5, the optimum replenishment time t_1^* decreases from 4.61 to 4.59, the optimal selling price s^* increased from 29.902 to 34.85, the optimum quantity order Q^* decreases from 20.306 to 20.231, the total profit P^* decreases from 422.25 to 421.998. When the holding cost ‘C_1’ decreases from 10 to 8.5, the optimum replenishment time t_1^* decreases from 4.61 to 3.99 and the total profit P^* decreases from 422.25 to 391.012, the optimum selling price s^* increased from 29.902 to 30.616, the optimum quantity order Q^* decreases from 20.306 to 19.95.

As replenishment parameter 'λ' increased from 5 to 5.75 units, the optimum replenishment time decreases from 4.61 to 3.326, optimum selling price increased from 29.902 to 35.664 the optimum quantity order Q^* increased from 20.306 to 21.309 and the total profit decreases from 422.25 to 319.985. As deteriorating parameter 'α' increased from 0.5 to 0.575, the optimum replenishment time decreases from 4.61 to 4.495, the optimum selling price decreases from 29.902 to 28.775, the optimum quantity order Q^* increased from 20.306 to 22.474, the total profit decreases from 422.25 to 413.804.

The demand parameter 'a' increased from 25 to 28.75 units, the optimum value of t_1^* increased from 4.61 to 5.946, the optimum value of s^* increased from 29.902 to 31.135, the optimum value of Q^* increased from 20.902 to 29.728, the total profit P^* decreases from 422.25 to 365.349. Another demand parameter 'b' increased from 0.5 to 0.575, the optimum replenishment time increased from 4.61 to 5.176, the optimum selling price decreases from 29.902 to 22.487, the optimum quantity order increased from 20.902 to 25.878, the total profit decreases from 422.25 to 314.179.

2.10. SENSITIVITY ANALYSIS OF THE MODEL:

The sensitivity analysis is conducted to investigate the impact of changes in model parameters and costs on optimum policies by varying each parameter (-15%, -10%,-5%, 0%, 5%, 10%, 15%) at a time for the model being studied. Table 2.4 summarizes the findings. Figure 2.4 shows the relationship between the parameters and the replenishment schedule's optimum values.

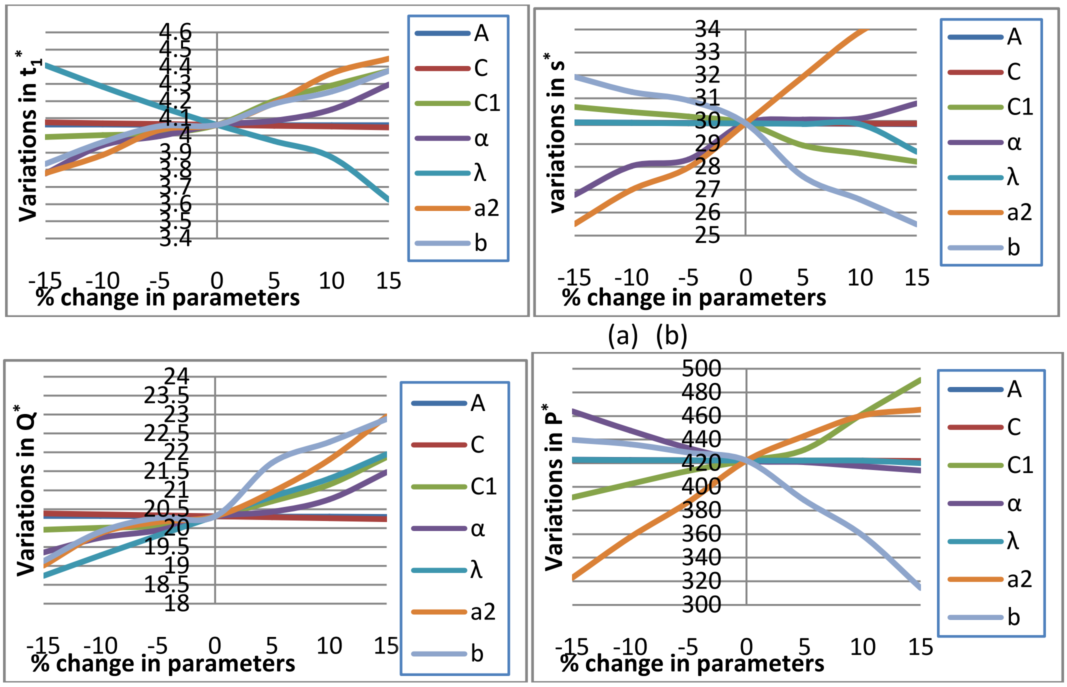
It is observed that the costs affect the optimum replenishment quantity and replenishment schedules significantly. As ordering cost A decreases, the optimum replenishment time t_1^*, the optimum quantity order Q^*, the optimum selling price s^* and total profit P^*increased. As ordering cost A increased, the optimum replenishment time t_1^*, the optimum quantity order Q^*, the optimum selling price s^* and total profit P^*decreases.

When the cost per unit C decreases, the optimum replenishment time t_1^*, the optimum quantity order Q^*, the optimum selling price s^* and total profit P^*increased. When the cost per unit C increased, the optimum replenishment time t_1^*, the optimum quantity order Q^*, the optimum and total profit P^*decreases. When the holding cost 'C_1' decreases, the optimum replenishment time t_1^*, the optimum quantity order Q^* are increased and total profit P^*, the optimum selling price s^* are decreases, when the holding cost 'C_1' increased, the optimum replenishment time t_1^*, the optimum quantity order Q^* and total profit P^* are increases and the optimum selling price s^* decreases.

Table 2.4
Analysis of model sensitivity - without shortages

Parameters	**optimum policies**	**Change of parameters**						
		-15%	**-10%**	**-5%**	**0%**	**5%**	**10%**	**15%**
A	t_1^*	4.064	4.063	4.062	4.061	4.06	4.06	4.059
	s^*	29.944	29.93	29.916	29.902	29.887	29.873	29.859
	Q^*	20.318	20.314	20.31	20.306	20.302	20.298	20.294
	P^*	423.075	422.8	422.525	422.25	421.975	421.7	421.425
C	t_1^*	4.076	4.071	4.066	4.061	4.056	4.051	4.046
	s^*	29.916	29.911	29.907	29.902	29.897	29.892	29.887
	Q^*	20.382	20.357	20.331	20.306	20.281	20.256	20.231
	P^*	422.507	422.422	422.336	422.25	422.163	422.076	421.988
C_1	t_1^*	3.99	4.001	4.011	4.061	4.201	4.29	4.375
	s^*	30.616	30.397	30.195	29.902	28.227	28.585	28.941
	Q^*	19.95	20.005	20.057	20.306	20.704	21.149	21.876
	P^*	391.012	402.298	413.505	422.25	431.443	461.817	490.38
α	t_1^*	3.778	3.942	3.996	4.061	4.086	4.151	4.295
	s^*	26.767	28.031	28.325	29.902	30.061	30.115	30.775
	Q^*	19.351	19.74	19.94	20.306	20.429	20.753	21.174
	P^*	463.785	447.517	432.619	422.25	420.922	417.308	413.804
λ	t_1^*	4.408	4.283	4.168	4.061	3.967	3.874	3.626
	s^*	29.924	29.915	29.908	29.902	29.869	29.864	28.664
	Q^*	18.735	19.275	19.798	20.306	20.828	21.309	21.95
	P^*	422.623	422.481	422.359	422.25	422.154	422.067	419.985
a	t_1^*	3.78	3.886	4.031	4.061	4.19	4.361	4.446
	s^*	25.495	26.992	27.975	29.902	31.934	33.887	35.135
	Q^*	19.001	19.861	20.157	20.306	20.948	21.806	22.95
	P^*	323.061	357.405	387.182	422.25	443.006	460.449	465.349
b	t_1^*	3.834	3.961	4.053	4.061	4.184	4.254	4.376
	s^*	31.929	29.078	31.894	29.902	27.578	26.568	25.487
	Q^*	19.125	19.904	20.267	20.306	21.919	22.27	22.878
	P^*	439.749	436.092	428.971	422.25	388.402	358.672	314.179

As replenishment parameter 'α' decreases, the optimum values of s^* and P^* are decreases, the optimum replenishment time t_1^* and the optimum selling price s^* are increases. As replenishment parameter 'α' increased, the optimum values of s^* and P^* are increases, the optimum replenishment time t_1^* and the optimum selling price s^* are decreases. As deteriorating parameter 'λ' decreases, the optimum replenishment time t_1^*, the optimum selling price s^* and total profit P^* are increases and the optimum quantity order Q^* is decreases. As deteriorating parameter 'γ' increased, the optimum replenishment time t_1^*, the optimum selling price s^* and total profit P^* are decreases and the optimum quantity order Q^* is increased.



(a) (b)

(c) (d)

Fig 2.4: Relationship between parameters and optimum values without shortages.

The demand parameter ‘a’ decreases, the optimum value of t_1^* is increased and the optimum values of s^* $Q^*$and P^* are decreasing. The demand parameter ‘a’ increased, the optimum value of t_1^* is decrease and the optimum values of s^* $Q^*$and P^* are increases. As another demand parameter ‘b’ decreases, the optimum values of t_1^*, s^* and Q^* are increases and the optimum value of P^* decreases. As another demand parameter ‘b’ increased, the optimum values of t_1^*, s^* and Q^* are decreasing and the optimum value of P^* increased.

CHAPTER III

INVENTORY MODEL FOR DETERIORATING ITEMS WITH EXPONENTIAL REPLENISHMENT AND PARETO DECAY HAVING TIME DEPENDENT DEMAND

INVENTORY MODEL FOR DETERIORATING ITEMS WITH EXPONENTIAL REPLENISHMENT AND PARETO DECAY HAVING TIME DEPENDENT DEMAND

3.1. INTRODUCTION:

This chapter contributes to the analysis and development of economic production quantity models for products that deteriorate with random production and decline in Pareto with time-dependent demand. Much of the literature has been published EPQ models for the deterioration of items with various assumptions on demand, rate of deterioration and production. For developing inventory models characterization of the commodity's lifetime with a probability distribution is required. To ascribe the distribution of probability to the commodity's lifetime, one must consider the commodity's embedded lifetime process. Ghare and Schrader (1963), Giri and Chadhuri (1999) assumed that an exponential distribution matches the lifetime of the product. Venkata Subbaiah, et al. (2004) assumed distribution of gamma throughout the lifetime of the goods. Mishra, et al. (2011), Skouri, et al. (2009) and Tadikamalla (1978) assumed distribution of Weibull throughout the lifetime of the products. Srinivasa Rao, et al. (2015) analyzed inventory models with Pareto lifetime. Madhavi, et al. (2008) developed random-life stock models. But all these authors assumed that the replenishment rate is infinite and it is instantaneous.

Many others developed finite replenishment inventory models away from the infinite replenishment rate (production). Goyal and Giri (2003) and Hu and Liu (2010) developed models of economic production with a constant rate of replenishment. Srinivasa Rao, et al. (2010) developed two different production rates were considered in one inventory system. Venkata Subbaiah, et al. (2011) has developed stock models of production level with alternating replenishment rate. Essay and Srinivasa Rao (2012) has developed stock-dependent inventory models.

However, the rate of production is not constant or uniform in many manufacturing or production processes will have a variable rate of production. The production is to be considered as random due to various random factors such as transportation, raw materials, environment, skill levels, tool wear etc, are influencing the production process. This situation is evident in areas where the product is perishable, such as food processing industries, chemical factories, cement industries, etc. Very little work in the literature has been published regarding EPQ models With production (replenishment) at random except the models of Sridevi, et al. (2010) and Srinivasa Rao, et al. (2010) who have inventory models have been developed with random replenishment and constant deterioration rate and also Lakshmana

Rao and Srinivasa Rao (2016) who have developed model EPQ with random replenishment and deterioration rate variable. But in a lot of commodities the commodity's lifetime is random and has a minimum threshold period to start deterioration. Therefore, characterizing the commodity's lifetime with a Pareto distribution is reasonable. Hence, in this paper we create and analyze some randomly generated EPQ models with Pareto decay with demand pattern depending on time.

3.2. ASSUMPTIONS:

i) The Power of demand is the function of time which is $f(t) = \frac{dt^{1/n}}{nT^{1/n}}$ Where ' n ' is the parameter of indexing ' T ' is the length of the cycle and total demand is 'd '.

ii) The production (replenishment) is finite and fits the density function of the Exponential distribution $f(t) = \lambda e^{-\lambda t}, t > 0, \lambda > 0$

Therefore the instantaneous replenishment is $k(t) = \frac{f(t)}{1-F(t)} = \lambda,\ \lambda > 0$

(iii) Leading period is zero

(iv) The length of the cycle T is fixed and known

(v) The shortages are allowable and completely backlogged

(vi) The deterioration unit has been lost

(vii) Instant deterioration rates is $h(t) = \frac{\alpha}{t}, t > 0$.

Notations:

The following observations are used for development of model

Q: Order the quantity in one cycle

A: Cost of ordering

C: Per unit cost

C_1: The cost of the inventory per unit of time

C_2: The cost of shortage per unit time

3.3. INVENTORY MODEL WITH SHORTAGES:

Consider the system of inventories where the inventory rate at time t=0 is zero. Inventory levels increase over the time $(0, t_1)$ due to additional demand replenishment and deterioration is met. When the inventory rate exceeds S, the replenishment ceases at time t_1. The stock is gradually decreasing due to demand and interval degradation (t_1, t_2). At t_2 the stock must generate null and back orders over the duration (t_2, t_3). At time t_3, after meeting the demand, the replenishment begins again and fulfills the backlog. The back orders are met during (t_3,T) and the stock level at the close of round T hits zero. The diagram schematic showing the instantaneous state of stock is shown in Figure 3.1

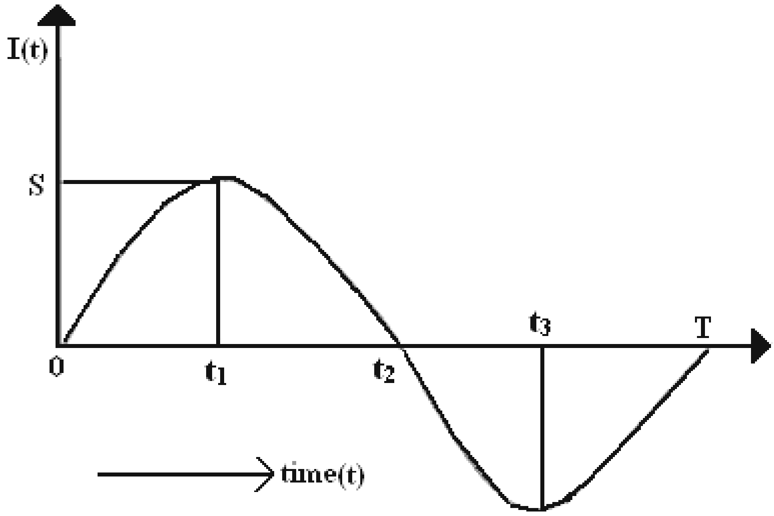


Fig 3.1: Inventory-level schematic diagram.

Let I(t) be the system's inventory at ' t ' time (0≤t≤T). Differential equations governing the instant I(t) status over the T phase duration.

$$\frac{d}{dt}I(t)+\frac{\alpha}{t}I(t)=\lambda-\frac{dt^{1/n}}{nT^{1/n}},0\leq t\leq t_1 \tag{3.3.1}$$

$$\frac{d}{dt}I(t)+\frac{\alpha}{t}I(t)=-\frac{dt^{1/n}}{nT^{1/n}},t_1\leq t\leq t_2 \tag{3.3.2}$$

$$\frac{d}{dt}I(t)=-\frac{dt^{1/n}}{nT^{1/n}},t_2\leq t\leq t_3 \tag{3.3.3}$$

$$\frac{d}{dt}I(t)=\lambda-\frac{dt^{1/n}}{nT^{1/n}},t_3\leq t\leq T \tag{3.3.4}$$

With the initial conditions I(0) = 0, I(t1) = S, I(t2) = 0 and I(T) = 0 and the differential equations are solved, the inventory on hand is obtained as ' t ' at the time.

$$I(t)=\frac{\lambda}{\alpha+1}\left(\frac{t^{\alpha+1}-t_1^{\alpha+1}}{t^{\alpha}}\right)+\frac{d}{T^{1/n}(1+\alpha n)}\left(\left(\frac{t_1}{t}\right)^{\alpha}t_1^{1/n}-t^{1/n}\right)+S\left(\frac{t_1}{t}\right)^{\alpha},0\leq t\leq t_1 \tag{3.3.5}$$

$$I(t)=\frac{-d}{T^{1/n}(1+\alpha n)}\left(\frac{t^{\alpha+1}-t_1^{\alpha+1}}{t^{\alpha}}\right)+S\left(\frac{t_1}{t}\right)^{\alpha},t_1\leq t\leq t_2 \tag{3.3.6}$$

$$I(t)=\frac{d}{T^{1/n}}\left(t_2^{1/n}-t^{1/n}\right),t_2\leq t\leq t_3 \tag{3.3.7}$$

$$I(t)=\lambda(t-T)+\frac{d}{T^{1/n}}\left(T^{1/n}-t^{1/n}\right),\ t_3\leq t\leq T \tag{3.3.8}$$

Loss of stock due to deterioration of the range (0, t)

$$L(t)=\int_0^t k(t)dt-\int_0^t \mathrm{f(t)}dt-I(t),\quad 0\leq t\leq t_2$$

$$L(t) = \begin{cases} \lambda t - \frac{d}{T^{1/n}} - \frac{\lambda}{\alpha+1}\left(\frac{t^{\alpha+1}-t_1^{\alpha+1}}{t^{\alpha}}\right) - \frac{d}{T^{1/n}}\left(\frac{t_1^{\alpha+1}-t^{\alpha+1}}{t^{\alpha}}\right) - S\left(\frac{t_1}{t}\right)^{\alpha}; 0 \le t \le t_1 \\ \lambda t_1 - \frac{d}{T^{1/n}} + \frac{d}{T^{1/n}(1+\alpha n)}\left(\frac{t^{\alpha+1}-t_1^{\alpha+1}}{t^{\alpha}}\right) + S\left(\frac{t_1}{t}\right)^{\alpha};\ t_1 \le t \le t_2 \end{cases}$$

Loss of stock due to deterioration of the T-length cycle

$$L(T) = \lambda t_1 - \mathrm{f(t)}t_2$$

The order quantity Q for the length cycle T is

$$Q = \int_0^{t_1} k(t)dt + \int_{t_3}^{T} k(t)dt$$

$$= \lambda(t_1 + T - t_3) \tag{3.3.9}$$

From equation (3.3.5) and to use the initial conditions I (0) = 0, we get the value of 'S '

$$S = \frac{\lambda}{\alpha+1} t_1 - \frac{d}{T^{1/n}(1+\alpha n)} t_1^{1/n} \tag{3.3.10}$$

when t = t_3, then equations (3.3.7) and (3.3.8) becomes

$$I(t) = \frac{d}{T^{1/n}}\left(t_2^{1/n} - t_3^{1/n}\right)$$

And $I(t) = \lambda(t_3 - T) + \frac{d}{T^{1/n}}\left(T^{1/n} - t_3^{1/n}\right)$

It is possible to compare the equations and simplify them

$$t_2 = T\left(\frac{\lambda}{d}(t_3 - T) + 1\right)^{n} \tag{3.3.11}$$

Let $K(t_1, t_{2\,,}t_3)$ be the total cost. Since the total cost is the amount of the cost collection, the cost of the items, the cost of keeping the stock, the total cost is

$$K(t_1, t_{2,}t_3) = \frac{(A)}{T} + \frac{(CQ)}{T} + \frac{h}{T}\left(\int_0^{t_1} I(t)dt + \int_{t_1}^{t_2} I(t)dt\right) + \frac{\pi}{T}\left(\int_{t_2}^{t_3}(-I(t))dt + \int_{t_3}^{T}(-I(t))dt\right) \tag{3.3.12}$$

Replacing the values of I(t) and Q in equation (3.3.12) K(t_1,t_2, t_3) can be obtained as

$$K(t_1, t_2, t_3) = \frac{A}{T} + \frac{C}{T}\lambda(\, t_1 + T - t_3) + \frac{C_1}{T}\left[\int_0^{t_1}\left(\frac{\lambda}{\alpha+1}\left(\frac{t^{\alpha+1}-t_1^{\alpha+1}}{t^{\alpha}}\right) + \frac{d}{T^{1/n}(1+\alpha n)}\left(\left(\frac{t_1}{t}\right)^{\alpha} t_1^{1/n} - \right.\right.\right.$$

$$\left.\left.\left. t^{1/n}\right) + S\left(\frac{t_1}{t}\right)^{\alpha}\right)dt + \int_{t_1}^{t_2}\left(\frac{-d}{T^{1/n}(1+\alpha n)}\left(\frac{t^{\alpha+1}-t_1^{\alpha+1}}{t^{\alpha}}\right) + S\left(\frac{t_1}{t}\right)^{\alpha}\right)dt\right]$$

$$+\frac{C_2}{T}\left[\int_{t_2}^{t_3}\left(\frac{d}{T^{1/n}}\left(t_2^{1/n} - t^{1/n}\right)\right)dt + \int_{t_3}^{T}\left(\lambda(t-T) + \frac{d}{T^{1/n}}\left(T^{1/n} - t^{1/n}\right)\right)dt\right] \tag{3.3.13}$$

On integration and simplification one can get

$$K(t_1, t_2, t_3) = \frac{A}{T} + \frac{C}{T}\lambda(\, t_1 + T - t_3) + \frac{C_1}{T}\left[\frac{\lambda}{\alpha+1}\left(\frac{t_1^2}{2} - \frac{t_1^2}{1-\alpha}\right) + \frac{d}{T^{1/n}(1+\alpha n)}\left(\frac{t_1}{1-\alpha} + \frac{t_1^{\alpha+1/n}t_2^{1-\alpha}}{1-\alpha}\right.\right.$$

$$-\frac{t_1^{1+1/n}}{1-\alpha}-\frac{n}{n+1}t_2^{1+1/n}\Bigg)+\frac{St_1^{\alpha}t_2^{1-\alpha}}{1-\alpha}\Bigg]+\frac{C_2}{T}\left[\frac{d}{T^{1/n}}\left(\frac{1}{n+1}\left(t_2^{1+1/n}-T^{1+1/n}\right)-t_3t_2^{1/n}+T^{1/n}t_3\right)\right.$$

$$\left.+\lambda\left(\frac{T^2}{2}-Tt_3+\frac{t_3^2}{2}\right)\right] \qquad (3.3.14)$$

Replacing the value of 'S ' in equation (3.3.10) in the overall cost equation (3.3.14)

$$K(t_1,t_2,t_3)=\frac{A}{T}+\frac{C}{T}\lambda(\,t_1+T-t_3)+\frac{C_1}{T}\left[\frac{\lambda}{\alpha+1}\left(\frac{t_1^2}{2}-\frac{t_1^2}{1-\alpha}+\frac{t_1^{\alpha+1}t_2^{1-\alpha}}{1-\alpha}\right)\right.$$

$$\left.+\frac{d}{T^{1/n}(1+\alpha n)}\left(\frac{t_1}{1-\alpha}-\frac{t_1^{\alpha+1/n}}{1-\alpha}-\frac{n}{n+1}t_2^{1+1/n}\right)\right]+\frac{c_2}{T}\left[\left(\frac{d}{T^{1/n}}\frac{1}{n+1}\left(t_2^{1+1/n}-T^{1+1/n}\right)\right)\right.$$

$$\left.-t_3t_2^{1/n}+T^{1/n}t_3\right)+\lambda\left(\frac{T^2}{2}-Tt_3+\frac{t_3^2}{2}\right)\Bigg] \qquad (3.3.15)$$

We obtain a replace for the value of ' t$_2$ ' in equation (3.3.11) in the total cost formula (3.3.15) is

$$K(t_1,t_3)=\frac{A}{T}+\frac{C}{T}\lambda(\,t_1+T-t_3)+\frac{C_1}{T}\left[\frac{\lambda}{\alpha+1}\left(\frac{t_1^2}{2}-\frac{t_1^2}{1-\alpha}+\frac{t_1^{\alpha+1}T^{1-\alpha}}{1-\alpha}\left(\frac{\lambda}{d}(t_3-T)+1\right)^{n(1-\alpha)}\right)\right.$$

$$\left.+\frac{d}{T^{1/n}(1+\alpha n)}\left(\frac{t_1}{1-\alpha}-\frac{t_1^{\alpha+1/n}}{1-\alpha}-\frac{n}{n+1}T^{1+1/n}\left(\frac{\lambda}{d}(t_3-T)+1\right)^{n+1}\right)\right]+\frac{c_2}{T}\left[d\left(\frac{T}{n+1}\left(\frac{\lambda}{d}(t_3-T)+1\right)^{n+1}-1\right)\right.$$

$$\left.+\lambda\left(\frac{T^2}{2}-\frac{t_3^2}{2}\right)\right] \qquad (3.3.16)$$

3.4. OPTIMAL PRICING AND POLICIES OF THE MODEL:

We obtain the optimum stock process policies under review in this chapter. We obtain the first order partial derivatives of K(t$_1$,t$_3$) given in equation (3.3.16) with respect to t$_1$ and t$_2$ and compare them to zero in order to find the optimal values of t$_1$ and t$_3$. The K(t$_1$,t$_3$) minimization state is

$$D=\begin{vmatrix}\dfrac{\partial^2K(t_1,t_3)}{\partial t_1^2} & \dfrac{\partial^2K(t_1,t_3)}{\partial t_1\partial t_3}\\ \dfrac{\partial^2K(t_1,t_3)}{\partial t_3\partial t_1} & \dfrac{\partial^2K(t_1,t_3)}{\partial {t_3}^2}\end{vmatrix}>0$$

Differentiating equation (3.3.16) to t$_1$ and to zero can be obtained

$$\frac{C\lambda}{T}+\frac{C_1}{T}\left[\frac{\lambda}{\alpha+1}\left(t_1-\frac{2t_1}{1-\alpha}+\frac{\alpha+1}{1-\alpha}t_1^{\alpha}T^{1-\alpha}\left(\frac{\lambda}{d}(t_3-T)+1\right)^{n(1-\alpha)}\right)+\frac{d}{T^{1/n}(1+\alpha n)}\left(\frac{1}{1-\alpha}-\right.\right.$$

$$\left.\left.\frac{n+1}{n(1-\alpha)}t_1^{1/n}\right)\right]=0 \qquad (3.4.1)$$

Differentiating equation (3.3.16) to t$_3$ and to zero can be obtained

$$\frac{C\lambda}{T} + \frac{C_1}{T}\left[t_1^{\alpha+1} T^{1-\alpha} n \frac{\lambda^2}{d(\alpha+1)}\left(\frac{\lambda}{d}(t_3 - T) + 1\right)^{n(1-\alpha)-1} - \frac{dnT^{1+1/n}}{T^{1/n}(1+\alpha n)}\frac{\lambda}{d}\left(\frac{\lambda}{d}(t_3 - T) + 1\right)^{n}\right]$$

$$+\frac{c_2}{T}\left[\lambda T\left(\left(\frac{\lambda}{d}(t_3 - T) + 1\right) +^{n} 1\right) - t_3\right] = 0 \tag{3.4.2}$$

Simultaneously solving equations (3.4.1) and (3.4.2) we obtain the optimum time at which replenishment is to be stopped t_1* of t_1 and the optimum time t_1* of t_3 at which replenishment is to be restarted after backorders accumulation.

The quantity of ordering Q* of Q in the period cycle T is obtained by replacing the optimal values of t_1*, t_3* in equation (3.3.9) as

$$Q^* = \lambda(t_1^* + T - t_3^*) \tag{3.4.3}$$

3.5. NUMERICAL ILLUSTRATION:

In this section we address the model's solution method through a statistical example by obtaining a stock system's replenishment uptime, replenishment downtime, order quantity and total cost of an inventory. Here, it believed that the product would deteriorate in nature and that shortages would be allowed and fully logged back. The values of the parameters and costs associated with the model are used to illustrate the solution process of the model:

α = 0.5, 0.525, 0.55, 0.575, A = 1000, 1500, 1100, 1150; C = 10, 10.5, 11, 11.5,
C_1 = 20, 21, 22, 23; C_2= 0.5, 0.525, 0.55, 0.575, λ= 5, 5.25, 5.5, 5.75, n= 2, 2.1, 2.2, 2.3,
d = 80, 84, 88, 92; T = Twelve months..

Substitute these values optimum quantity of order, replenishment uptime, replenishment down time and total cost are calculated and presented in Table 3.1.

From Table 3.1 shows that that parameters of deterioration and replenishment have a tremendous influence on optimum replenishment times, order size, and total cost.

Table 3.1

Optimum values of t_1^*, t_3^*, Q^* and K^* for different values of parameters

A	C	C_1	C_2	T	λ	α	n	d	t_1	t_3	Q	K
1000	**10**	**20**	**0.5**	**12**	**5**	**0.5**	**2**	**80**	1.124	4.217	44.536	74.317
1050				**12**					1.178	4.296	44.412	76.177
1100				**12**					1.22	4.368	44.260	78.198
1150				**12**					1.257	4.439	44.089	80.278
	10.5			**12**					1.14	4.252	44.436	75.189
	11.0			**12**					1.166	4.291	44.374	75.899
	11.5			**12**					1.18	4.325	44.274	76.784
		21		**12**					1.122	4.175	44.736	72.313
		22		**12**					1.12	4.136	44.916	70.303
		23		**12**					1.117	4.101	45.078	68.288
			0.525	**12**					1.133	4.229	44.516	74.734
			0.55	**12**					1.141	4.242	44.495	75.15
			0.575	**12**					1.15	4.255	44.474	75.565
				12	**5.25**				1.25	4.43	46.305	79.757
				12	**5.50**				0.82	4.666	44.842	85.951
				12	**5.75**				1.191	4.926	47.524	89.099
				12		**0.525**			1.175	4.239	44.681	74.76
				12		**0.550**			1.215	4.252	44.814	75.279
				12		**0.575**			0.841	4.319	42.612	76.739
				12			**2.1**		1.185	4.297	44.437	76.737
				12			**2.2**		0.803	4.391	42.055	80.051
				12			**2.3**		1.274	4.442	44.165	81.401
				12				**84**	0.997	4.012	44.924	67.223
				12				**88**	0.884	3.808	45.38	59.387
				12				**92**	0.802	3.613	45.945	50.811

If cost of ordering ‘A’ increases from 1000 to 1150, then optimum quantity of order Q^* decreases from 44.536 to 44.089, optimum downtime replenishment t_1^* increases from 1.124 to 1.257, optimum uptime replenishment t_3^* increases from 4.217 to 4.439 and total cost K^*, increases from 74.317 to 80.278. The cost parameter ‘C’ increases between 10 to 11.5 optimum quantity of order Q^* decreases from 44.536 to 44.274, optimum downtime replenishment t_1^* decreases from 1.124 to 1.18, optimum uptime replenishment t_3^* increases from 4.217 to 4.325, and total cost K^*, increases from 74.317 to 76.784.

When inventory holding cost ‘C_1’ increases from 20 to 23, optimum quantity of order Q^* increases from 44.536 to 45.078, optimum downtime replenishment t_1^* decreases from 1.124 to 1.117, optimum uptime replenishment t_3^* decreases from 4.217 to 4.101 and total cost K^*, decreases from 74.317 to 68.288. As shortage cost ‘C_2’ increases from 0.5 to 0.575, then optimum quantity of order Q^* decreases from 44.536 to 44.474, optimum downtime replenishment t_1^* decreases from 1.124 to 1.15, optimum uptime replenishment t_3^* increases from 4.217 to 4.255 and total cost K^*, increases from 74.317 to 89.099.

If replenishment parameter ‘λ’ increases between 5 to 5.75, then optimum quantity of order Q^* increases from 44.536 to 47.524, optimum downtime replenishment t_1^* increases from 1.124 to 1.191, optimum uptime replenishment t_3^* increases from 4.217 to 4.926 and total cost K^*, increases from 74.317 to 89.099. When deteriorating parameter ‘α’ increases between 0.5 to 0.575 optimum quantity of order Q^* decreases from 44.536 to 42.612, optimum downtime replenishment t_1^* decreases from 1.124 to 0.841, optimum uptime replenishment t_3^* increases from 4.217 to 4.319 and total cost K^*, increases from 74.317 to 76.739.

The indexing parameter ‘n’ increases between 2 to 2.3, optimum quantity of order Q^* decreases from 44.536 to 44.165, optimum downtime replenishment t_1^* increases from 1.124 to 1.274, optimum uptime replenishment t_3^* increases from 4.217 to 4.442 and the total cost K^*, increases from 74.317 to 81.401. As demand parameter‘d’ increases from 80 to 92, optimum quantity of order Q^* increases from 44.536 to 45.945, optimum downtime replenishment t_1^* decreases from 1.124 to 0.802, optimum uptime replenishment t_3^* decreases from 4.217 to 3.613 and total cost K^*, decreases from 74.317 to 50.811.

3.6. SENSITIVITY ANALYSIS OF THE MODEL:

The sensitivity analysis is carried out to analyze the effect on optimal policies of changes in process parameters and costs by varying the parameter at a time for the model being evaluated (-15%, -10%,-5%, 0%, 5%, 10%, 15%). The findings are shown in Table 3.2. Figure 3.2 shows the relationship between the optimum values and the parameters.

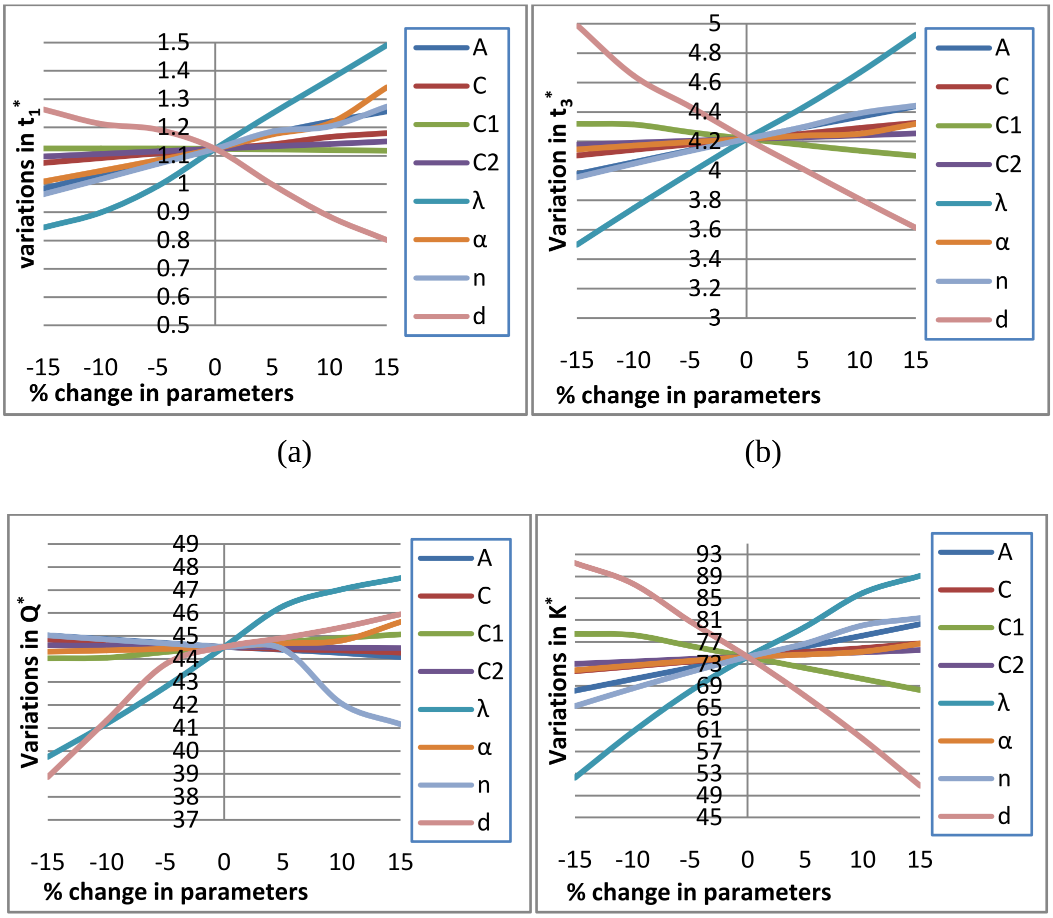
It is found that the costs affect the optimal order schedules of quantities and replenishment significantly. As cost of ordering A decreases, the optimum downtime replenishment t_1^*, optimum uptime replenishment t_3^* and total cost K^* are decreases and optimum quantity of order Q^* increases. As cost of ordering A increases, the optimum downtime replenishment t_1^*, optimum uptime replenishment t_3^* and total cost K^* are increases and optimum quantity of order Q^* decreases. When cost per unit C decreases, optimum uptime replenishment t_3^*, optimum downtime replenishment t_1^* and total cost K^* are decreases and optimum quantity of order Q^* increases. When cost per unit C increases, optimum uptime replenishment t_3^*, optimal downtime replenishment t_1^* and total cost K^* are increases and optimum quantity of order Q^* decreases.

When holding cost ‘C_1’ decreases, optimum uptime replenishment t_3^*, the optimum downtime replenishment t_1^* and total cost K^* are increases and optimum quantity of order Q^* decreases. When holding cost ‘C_1’ increases, optimum uptime replenishment t_3^*, optimum downtime replenishment t_1^* and total cost K^* are decreases and optimal quantity of order Q^* increases. If shortage cost ‘C_2’ decreases, then optimum uptime replenishment t_3^*, optimal downtime replenishment t_1^* and total cost K^* are decreases and optimum quantity of order Q^* increases. If shortage cost ‘C_2’ increases, then optimum uptime replenishment t_3^*, optimum downtime replenishment t_1^* and total cost K^* are increases and optimal quantity of order Q^* decreases.

Table 3.2
System Sensitivity analysis - with shortages

T Parameters	Optimum policies	Variations in parameters						
		-15%	**-10%**	**-5%**	**0%**	**5%**	**10%**	**15%**
A	t_1^*	0.984	1.032	1.079	1.124	1.178	1.22	1.257
	t_3^*	3.977	4.059	4.139	4.217	4.296	4.368	4.439
	Q^*	45.038	44.865	44.698	44.536	44.412	44.26	44.089
	K^*	68.165	70.235	72.286	74.317	76.177	78.198	80.278
C	t_1^*	1.074	1.091	1.108	1.124	1.14	1.166	1.18
	t_3^*	4.105	4.143	4.18	4.217	4.252	4.291	4.325
	Q^*	44.846	44.741	44.637	44.536	44.436	44.374	44.274
	K^*	71.681	72.563	73.442	74.317	75.189	75.899	76.784
C_1	t_1^*	1.125	1.125	1.125	1.124	1.122	1.12	1.117
	t_3^*	4.319	4.314	4.263	4.217	4.175	4.136	4.101
	Q^*	44.031	44.058	44.311	44.536	44.736	44.916	45.078
	K^*	78.502	78.303	76.314	74.317	72.313	70.303	68.288
C_2	t_1^*	1.097	1.106	1.115	1.124	1.133	1.141	1.15
	t_3^*	4.178	4.191	4.204	4.217	4.229	4.242	4.255
	Q^*	44.595	44.576	44.556	44.536	44.516	44.495	44.474
	K^*	73.065	73.483	73.901	74.317	74.734	75.15	75.565
λ	t_1^*	0.846	0.899	0.993	1.124	1.25	1.37	1.491
	t_3^*	3.497	3.746	3.985	4.217	4.43	4.666	4.926
	Q^*	39.736	41.187	42.789	44.536	46.305	44.842	47.524
	K^*	52.24	60.592	68.045	74.317	79.757	85.951	89.099
α	t_1^*	1.009	1.046	1.084	1.124	1.175	1.215	1.341
	t_3^*	4.146	4.172	4.195	4.217	4.239	4.252	4.319
	Q^*	44.316	44.371	44.444	44.536	44.681	44.814	45.612
	K^*	71.866	72.779	73.598	74.317	74.76	75.279	76.739
n	t_1^*	0.963	1.017	1.071	1.124	1.185	1.203	1.274
	t_3^*	3.956	4.048	4.135	4.217	4.297	4.391	4.442
	Q^*	45.038	44.844	44.68	44.536	44.437	42.055	41.165
	K^*	65.358	68.554	71.541	74.317	76.737	80.051	81.401
d	t_1^*	1.263	1.212	1.192	1.124	0.997	0.884	0.802
	t_3^*	4.991	4.647	4.437	4.217	4.012	3.808	3.613
	Q^*	40.863	40.325	43.778	44.536	44.924	45.38	45.945
	K^*	91.428	87.721	80.823	74.317	67.223	59.387	50.811

The replenishment parameter 'λ' decreases, optimum values t_3^*, t_1^*, $Q^*$and K^* are decreases. If replenishment parameter 'λ' increases then optimum values t_3^*, t_1^*, $Q^*$and K^* are increases. If deterioration parameter 'α' decreases then optimal uptime replenishment t_3^* increases, optimum downtime replenishment t_1^*, optimum quantity of order Q^* and total cost K^* are decreases. As deterioration parameter 'α' increases, optimum uptime replenishment t_3^* increases, optimum downtime replenishment t_1^*, optimum quantity of order Q^* and total cost K^* are increases.



(a) (b)

(c) (d)

Fig 3.2 : Relationship between parameters and optimum shortage values

If indexing parameter 'n' decreases, then optimum values t_3^*, t_1^*, $Q^*$and K^* are decreases. If indexing parameter 'n' increases, then optimum values t_3^*, t_1^*, $Q^*$and K^* are increases. When demand parameter 'd' decreases, optimum uptime replenishment t_3^*, optimum downtime replenishment t_1^* and total cost K^* are increases and optimum quantity of order Q^* decreases. When the demand parameter 'd' decreases, optimum uptime replenishment t_3^*, optimum downtime replenishment t_1^* and total cost K^* are increases and optimum quantity of order Q^* increases.

3.7. INVENTORY MODEL WITHOUT SHORTAGES:

In this section, the stock model is built and evaluated to deteriorate products without shortages. Here, it presumed that shortages are not allowed and that inventory rate at time t = 0 is zero. During the time (0, t_1) the inventory rate rises due to excess replenishment after demand fulfillment and deterioration. When the inventory rate exceeds S, the replenishment ends at time t_1. The stock is gradually decreasing due to demand and interval deterioration (t_1, T). The stock hits zero at the time T. The diagram showing the instantaneous stock status is shown in Figure 3.3

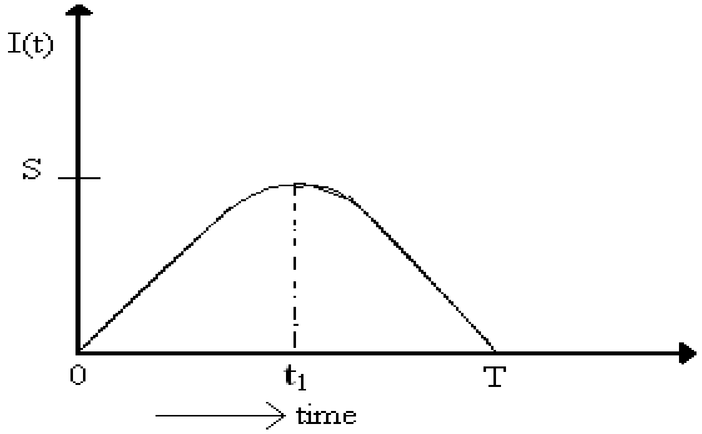


Fig 3.3: Schematic diagram showing the degree of the stocks.

Let I(t) be the inventory level of the system at ' t ' time (0≤t≤T). Differential equations that govern the instant state of I(t) over the duration of the T phase.

$$\frac{d}{dt}I(t)+\frac{\alpha}{t}I(t)=\lambda-\frac{d.t^{\frac{1}{n}-1}}{n.T^{\frac{1}{n}}};\qquad 0\le t\le t_1 \tag{3.7.1}$$

$$\frac{d}{dt}I(t)+\frac{\alpha}{t}I(t)=-\frac{d.t^{\frac{1}{n}-1}}{n.T^{\frac{1}{n}}};\qquad t_1\le t\le T \tag{3.7.2}$$

Using initial conditions, I(0) = 0, I(t_1) = S and I(T) = 0 and the differential equations are solved, the stock on hand is obtained as ' t ' at the time.

$$I(t)=\frac{\lambda}{\alpha+1}\left(t-\left(\frac{t_1}{t}\right)^{\alpha}t_1\right)+\frac{d}{T^{1/n}(1+\alpha n)}\left(\left(\frac{t_1}{t}\right)^{\alpha}{t_1}^{1/n}-t^{1/n}\right)+S\left(\frac{t_1}{t}\right)^{\alpha};0\le t\le t_1 \tag{3.7.3}$$

$$I(t)=\frac{d}{T^{1/n}(1+\alpha n)}\left(\left(\frac{t_1}{t}\right)^{\alpha}{t_1}^{1/n}-t^{1/n}\right)+S\left(\frac{t_1}{t}\right)^{\alpha};\ ;\ t_1\le t\le \mathrm{T} \tag{3.7.4}$$

Loss of stock due to interval deterioration (0, t)

$$L(t)=\int_0^t k(t)dt-\int_0^t \mathrm{f(t)}dt-I(t)\,,0\le t\le T$$

$$L(t) = \begin{cases} \lambda t - \dfrac{dt^{1/n}}{T^{1/n}} \dfrac{\lambda}{\alpha+1}\left(t - \left(\dfrac{t_1}{t}\right)^{\alpha} t_1\right) + \dfrac{d}{T^{1/n}(1+\alpha n)}\left(\left(\dfrac{t_1}{t}\right)^{\alpha} {t_1}^{1/n} - t^{1/n}\right) + S\left(\dfrac{t_1}{t}\right)^{\alpha}; & 0 \le t \le t_1 \\ \lambda t_1 + \dfrac{d}{T^{1/n}(1+\alpha n)}\left(\left(\dfrac{t_1}{t}\right)^{\alpha} {t_1}^{1/n} - t^{1/n}\right) + S\left(\dfrac{t_1}{t}\right)^{\alpha}; & t_1 \le t \le t_2 \end{cases}$$

Ordering quantity Q for the length cycle T is

$$Q = \int_0^{t_1} k(t)dt = \lambda t_1 \tag{3.7.5}$$

Using the initial condition I (0) = 0 from equation (3.7.3) we get the value of 'S ' as

$$S = \frac{\lambda}{\alpha+1} t_1 - \frac{d}{T^{1/n}(1+\alpha n)} {t_1}^{1/n} \tag{3.7.6}$$

Let K(t_1) be the total cost per time per unit. Because the total cost is the amount of the cost of set-up, the cost of items, the cost of keeping stock. The total cost of this is

$$K(t_1) = \frac{A}{T} + \frac{CQ}{T} + \frac{h}{T}\left(\int_0^{t_1} I(t)dt + \int_{t_1}^{T} I(t)dt\right) \tag{3.7.7}$$

We obtain K(t_1) as a substitute for the value of I (t) and Q given in equation (3.7.4), (3.7.5) and (3.7.6) as equation (3.7.7).

$$K(t_1) = \frac{A}{T} + \frac{C}{T}\lambda t_1$$

$$+\frac{C_1}{T}\left[\frac{\lambda}{\alpha+1}\int_0^{t_1}\left(t - \left(\frac{t_1}{t}\right)^{\alpha} t_1\right)dt + \frac{d}{T^{1/n}(1+\alpha n)}\int_0^{t_1}\left(\left(\frac{t_1}{t}\right)^{\alpha} {t_1}^{1/n} - t^{1/n}\right)dt + S\int_0^{t_1}\left(\frac{t_1}{t}\right)^{\alpha} dt\right.$$

$$\left. - \frac{d}{T^{1/n}(1+\alpha n)}\int_{t_1}^{T}\left(\left(\frac{t_1}{t}\right)^{\alpha} {t_1}^{1/n} - t^{1/n}\right)dt + S\int_{t_1}^{T}\left(\frac{t_1}{t}\right)^{\alpha} dt\right.$$

On integration and simplification one can get

$$K(t_1) = \frac{A}{T} + \frac{C}{T}\lambda t_1 + \frac{C_1}{T}\left[\frac{\lambda}{\alpha+1}\left(\frac{t_1^2}{2} - \frac{t_1^2}{1-\alpha}\right) + \frac{d}{T^{1/n}(1+\alpha n)}\left(\frac{t_1^{1+1/n}}{1-\alpha} - \frac{n t_1^{1+1/n}}{n+1}\right)\right.$$

$$\left. + \frac{S{t_1}^{\alpha}}{1-\alpha} - \frac{d}{T^{1/n}(1+\alpha n)}\left(\left(T^{1+1/n} - t_1^{1+1/n}\right)\left(\frac{n}{n+1} - \frac{1}{1-\alpha}\right)\right) + \frac{S{t_1}^{\alpha}}{1-\alpha}\left(T^{-\alpha+1} - t_1^{-\alpha+1}\right)\right] \tag{3.7.8}$$

Substituting the value of ‘S’ in given equation (3.7.6) in the cost equation (3.7.8), one can get

$$K(t_1) = \frac{A}{T} + \frac{C}{T}\lambda t_1 + \frac{C_1}{T}\left[\frac{\lambda}{\alpha+1}\left(\frac{t_1^2}{2} - \frac{2t_1^2}{1-\alpha} + \frac{t_1^2}{(1-\alpha)(1+\alpha)} + \frac{t_1^{\alpha+1}T^{-\alpha+1}}{(1-\alpha)(1+\alpha)}\right)\right.$$

$$\left. - \frac{d}{T^{1/n}(1+\alpha n)}\left(\frac{n}{n+1}T^{1+1/n} - \frac{T^{1+1/n}}{1-\alpha} + T^{-\alpha+1} t_1^{\alpha+1/n}\right)\right] \tag{3.7.9}$$

3.8. OPTIMAL PRICING AND POLICIES OF THE MODEL:

In this paper we get the best stock process policies that are being studied. In order to find the optimal values of t_1, we compare $K(t_1)$'s first order partial derivatives to zero within relation to t_1. The minimum requirement for $K(t_1)$ is

$$\frac{d^2K(t_1)}{dt_1^2} > 0$$

Differentiating $K(t_1)$ and equating to zero with respect to t_1

$$= \frac{\lambda}{\alpha+1}\left(t_1 - \frac{4t_1}{1-\alpha} + \frac{2t_1}{(1-\alpha)(1+\alpha)} + \frac{t_1T^{1-\alpha}}{1-\alpha}\right) - \left(\frac{d}{T^{1/n}(1+\alpha n)}\right)\left(\alpha + \frac{1}{n}\right) t_1^{\alpha+\frac{1}{n}-1} = 0 \qquad (3.8.1)$$

We obtain the optimal time to stop the replenishment at t_1^* of t_1 by solving the formula (3.8.1).

The optimum order quantity Q * of Q in process T is obtained by replacing the optimal value of t_1 in equation (3.7.5).

$$Q^* = \lambda t_1{}^* \qquad (3.8.2)$$

3.9. NUMERICAL ILLUSTRATION:

We address numerical examples in this paper. The values of the costs and parameters associated with the model are used to illustrate the solution process of the model:

A = 2000, 2100, 2200, 2300; C = 10, 10.5, 11, 11.5; C_1 = 10, 10.5, 11, 11.5;

α = 0.5, 0.525, 0.55, 0.575, λ = 5, 5.25, 5.5, 5.75; n = 2, 2.1, 2.2, 2.3;

d = 100, 105, 110, 115; T = Twelve months.

Optimum quantity of order Q*, replenishment time, total cost are estimated and provided in Table 3.3 to replace these values.

Table 3.3 shows that the decay and replenishment parameters have a tremendous impact on the optimum values of the model.

If cost of order ‘A’ increases from 2000 to 2300, then optimum quantity of ordering Q^* increases from 43.973 to 45.814, optimum time of replenishment t_1^* increases from 8.795 to 9.163 and the total cost K^*, decreases from 372.656 to 365.292. If cost parameter ‘C’ increases between 10 to 11.5 then optimum quantity of order Q^* increases from 43.973 to 45.052, optimum time of replenishment t_1^* increases from 8.795 to 9.01, and total cost K^*, decreases from 372.656 to 359.457. As holding cost ‘C_1’ increases between 10 to 11.5 optimum quantity of order Q^* decreases from 43.973 to 43.409, optimum time of replenishment t_1^* decreases between 8.795 to 8.682, and total cost K^*, increases from 372.656 to 409.26.

When parameter of replenishment ‘λ’ increases between 5 to 5.75 optimum quantity of order Q^* decreases from 43.973 to 40.321, optimum time of replenishment t_1^* decreases from 8.795 to 7.233, and the total cost K^*, increases from 372.656 to 402.87. The parameter of deteriorating ‘α’ increases between 0.5 to 0.575 optimum quantity of order Q^* decreases from 43.973 to 38.1, optimum time of replenishment t_1^* decreases from 8.795 to 7.458, and the total cost K^*, increases from 372.656 to 545.183.

If parameter of indexing ‘n’ increases between 2 to 2.3 then optimum quantity of order Q^* decreases from 43.973 to 36.126, optimum time of replenishment t_1^* decreases from 8.795 to 7.225, and the total cost K^*, increases from 372.656 to 448.35. As demand parameter ‘d’ increases from 100 to 115, then optimum quantity of order Q^* decreases from 43.973 to 37.367, optimum time of replenishment t_1^* decreases from 8.795 to 7.473, and the total cost K^*, increases from 372.656 to 553.67.

Table 3.3

Optimum t_1 *, Q * and K * values of various parameter values

A	**C**	**C_1**	**T**	**α**	**λ**	**n**	**d**	**t_1**	**Q**	**K**
2000	**10**	**10**	12	**0.5**	**5**	**2**	**100**	8.795	43.973	372.656
2100			12					8.919	44.597	370.147
2200			12					9.042	45.211	367.693
2300			12					9.163	45.814	365.292
	10.5		12					8.868	44.338	368.185
	11.0		12					8.939	44.697	363.785
	11.5		12					9.01	45.052	359.457
		10.5	12					8.754	43.77	384.8
		11.0	12					8.716	43.582	397.004
		11.5	12					8.682	43.409	409.26
			12	**0.525**				7.776	39.288	508.819
			12	**0.550**				7.62	38.88	526.19
			12	**0.575**				7.458	38.1	545.183
			12		**5.25**			8.019	41.348	392.819
			12		**5.50**			7.235	40.152	395.8
			12		**5.75**			7.233	40.321	402.87
			12			**2.1**		8.288	41.442	399.138
			12			**2.2**		7.764	38.82	424.529
			12			**2.3**		7.225	36.126	448.35
			12				**105**	8.253	41.263	450.409
			12				**110**	7.552	37.76	543.542
			12				**115**	7.473	37.367	553.67

3.10. SENSITIVITY ANALYSIS OF THE MODEL:

The analysis of sensitivity is conducted to investigate the effect on optimal policies of changes in model parameters and costs by changing each parameter (-15%, -10%,-5%, 0%, 5%, 10%, 15%) at a time for the model being studied. Table 3.4 summarizes the findings. Figure 3.4 shows the relationship between the parameters and the replenishment schedule's optimum values.

Table 3.4
Form Sensitivity testing – without shortages

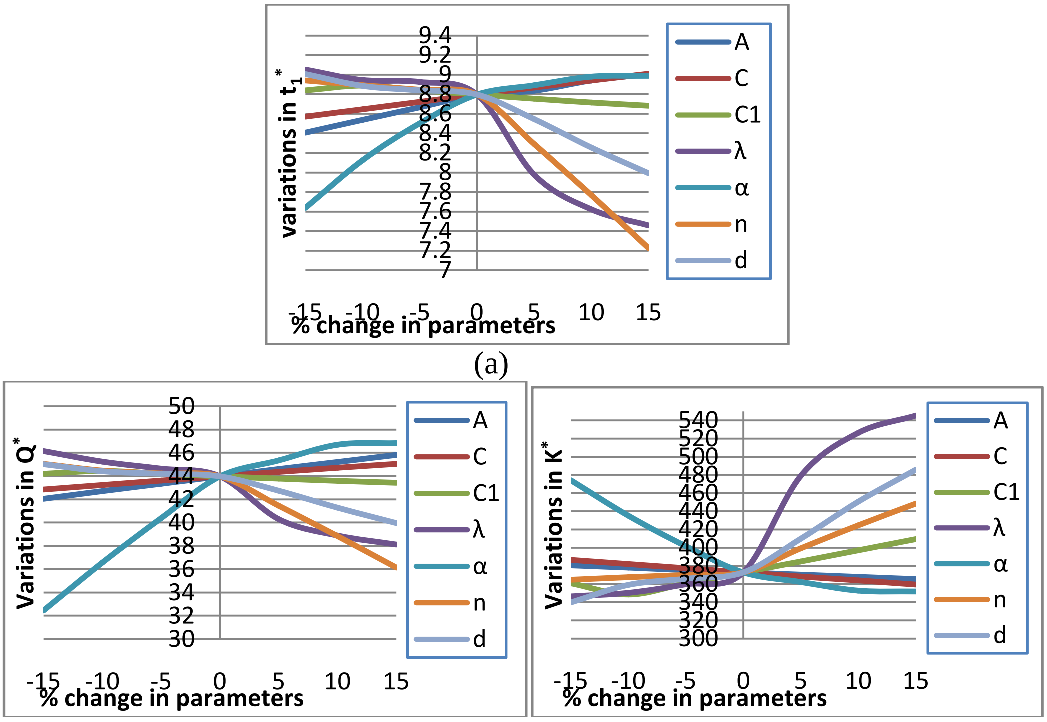
Parameters	**Optimum policies**	**Change in parameters**						
		-15%	**-10%**	**-5%**	**0%**	**5%**	**10%**	**15%**
A	t_1^*	8.406	8.538	8.667	8.795	8.919	9.042	9.163
	Q^*	42.03	42.69	43.337	43.973	44.597	45.211	45.814
	K^*	380.519	377.84	375.22	372.656	370.147	367.693	365.292
C	t_1^*	8.57	8.646	8.721	8.795	8.868	8.939	9.01
	Q^*	42.848	43.228	43.603	43.973	44.338	44.697	45.052
	K^*	386.492	381.81	377.197	372.656	368.185	363.785	359.457
C_1	t_1^*	8.839	8.887	8.839	8.795	8.754	8.716	8.682
	Q^*	44.193	44.433	44.193	43.973	43.77	43.582	43.409
	K^*	360.58	348.583	360.58	372.656	384.8	397.004	409.26
α	t_1^*	9.051	8.945	8.925	8.795	7.776	7.62	7.458
	Q^*	46.125	45.224	44.623	43.973	37.288	38.1	38.88
	K^*	346.239	350.358	359.556	372.656	508.819	526.19	545.183
λ	t_1^*	7.636	8.125	8.503	8.795	8.891	8.979	8.987
	Q^*	32.451	36.561	40.388	43.973	45.346	46.688	46.821
	K^*	474.066	435.126	401.54	372.656	362.283	352.53	351.588
n	t_1^*	8.942	8.893	8.844	8.795	8.288	7.764	7.225
	Q^*	44.71	44.466	44.22	43.973	41.442	38.82	36.126
	K^*	364.592	367.283	369.971	372.656	399.138	424.529	448.35
d	t_1^*	9.005	8.884	8.84	8.795	8.544	8.253	7.991
	Q^*	45.025	44.418	44.199	43.973	42.718	41.263	39.957
	K^*	339.737	359.008	365.752	372.656	409.564	450.409	485.862

It is observed that the costs affect the optimum quantity of order and replenishment schedules significantly. As cost of order A decreases, then optimum time of replenishment t_1^* and optimum quantity of order Q^* are decreases and total cost K^* increases. As cost of order A increases, then optimum time of replenishment t_1^* and the optimum quantity of order Q^* are increases and the total cost K^* decreases. As cost per unit C decreases, optimum time of replenishment t_1^* and optimum quantity of order Q^* are decreases and total cost K^* increases. As cost per unit C increases, optimum time of replenishment t_1^* and optimum quantity of order Q^* are increases and total cost K^* decreases. When holding cost 'C_1' decreases, optimum time of replenishment t_1^* and optimum quantity of order Q^* are increases and the total cost K^* decreases. When holding cost 'h' increases, optimum time of replenishment t_1^* and optimum quantity of order Q^* are decreases and total cost K^* increases.

If replenishment parameter 'α' decreases, then optimum time of replenishment t_1^* and optimum ordering quantity Q^* are increases and total cost per unit time K^* decreases. If parameter of replenishment 'α' increases, then optimum time of replenishment t_1^* and optimum ordering quantity Q^* are decreases and total cost per unit time K^* increases. When parameter of deterioration 'λ' decreases, optimum time of replenishment t_1^* and optimum ordering quantity Q^* are decreases and total cost per unit time K^* increases. When parameter of deterioration 'λ' increases, optimum time of replenishment t_1^* and optimum ordering quantity Q^* are increases and total cost K^* decreases.

As parameter of indexing 'n' decreases, optimum values of t_1^* Q^* are increases and total cost K^* decreases. As parameter of indexing 'n' increases, optimum values of t_1^* Q^* are decreases and total cost K^* increases. If demand parameter 'd' decreases, optimum time of replenishment t_1^* and optimum quantity of order Q^* are increases and the total cost K^* decreases. If parameter of demand 'd' increases, optimum time of replenishment t_1^*, optimum quantity of order Q^* are decreases and total cost K^* increases.

It is also noted that the shortage inventory model's optimum total cost is less than the shortage-free inventory model. If the demand is a function of time, enabling back-logged shortages is rational. Historical data are generated by managers by estimating demand parameters, replenishment parameters, and deteriorating parameters to obtain optimal supply process policies. The framework also incorporates some existing models as specific cases when the replenishment distribution degenerates.



(a)

(b) (c)

Fig 3.4: Relationship between optimal values and parameters with shortages

CHAPTER – IV

SUMMARY AND CONCLUSIONS

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REFERENCES

SUMMARY AND CONCLUSIONS

This project contributes to the development and analysis of economic production quantity (EPQ) models for deteriorating items with random production and Pareto decay having different types of demand. The EPQ models are mathematical models which represent the inventory situation in a production or manufacturing system. The EPQ models can also be utilized for scheduling the optimal operating policies of market yards, warehouses, godowns, etc. In many of the inventory models the replenishment and production are considered synonymously. The economic production quantity models provide optimal decisions regarding the quantity to be produced (to be ordered) the production downtime and the production uptime. The EPQ models can be categorized into two categories namely, i) EPQ models for deteriorating items and ii) EPQ models for infinite lifetime. The EPQ models for deteriorating items gained lot of importance for the last two decades due to their ready applicability.

Much work has been reported in literature regarding EPQ models for deteriorating items with various assumptions on demand, rate of deterioration and production. For developing inventory models it is required to characterize the life time of the commodity with a probability distribution. For ascribing the probability distribution to the life time of the commodity, one has to consider the embedded process of the lifetime of the commodity. Ghare and Schrader (1963), Cohen (1977), Aggrawal (1978), Dave and Shah (1982), Pal (1990), Jamal, et al. (1997) and Giri and Chaudhuri (1999) assumed that the lifetime of the commodity follows an exponential distribution. Tadikamalla (1978) assumed Gamma distribution to the lifetime of the commodity. Covert and Philip(1973), Philip (1974), Goel and Aggrawal (1980), Hwang and Choi (1984), Venkat Subbaiah, et al. (2004), Skouri, et al. (2009) and Mishra, et al. (2011) assumed Weibull distribution to the lifetime of the commodity. Nirupama Devi (2001) developed inventory models with mixtures of Weibull distribution. Srinivasa Rao, et al. (2005) developed inventory models with generalized Pareto lifetime. Madhavi, et al. (2008) developed inventory models with random lifetime. But all these authors assumed that the replenishment is instantaneous with infinite rate.

Away from the infinite rate of replenishment (production), several others developed inventory models with finite rate of replenishment. Mandal and Phaujdar (1989), Goyal and Gunasekhran (1995), Jiyang and Du (1998), Goyal and Giri (2003), Sana et al. (2004), Gantg and Wang (2005), Lin et al. (2006), Maiti et al. (2007), Hu and Liu (2010) and Uma Maheswara Rao, et al. (2010) have developed economic production quantity models with constant rate of replenishment. Perumal and Srivarignan (2002) considered two different rates of production in one inventory system. Pal and Mandal (1997), Sen and Chakrabarthy (2007) and Venkat Subbaiah, et al. (2011) have developed production level inventory models with alternating rate of replenishment. Mahata and Goswami (2006) considered fuzzy production rates for developing inventory models. Essay, et al. (2012) has developed inventory models with stock dependent demand.

But in many production or manufacturing systems the production rate is not constant or uniform, will have a variable rate of production. The production is to be considered as random due to various random factors such as transportation, raw materials, environment, skill levels, tool wear etc, are influencing the production process. This scenario is visible at places like food processing industries, chemical industries, cement industries, etc. where the commodity is perishable. Very little work has been reported in literature regarding EPQ models with random production (replenishment) except the models of Sridevi, et al. (2010), Srinivasa, Rao et al. (2010), who have developed inventory models with random replenishment and constant rate of deterioration.

But in many commodities the life time of the commodity is random and has a minimum threshold period to start deterioration. Hence, it is reasonable to characterize the lifetime of the commodity with a Pareto distribution. Hence, in this thesis we develop and analyze some EPQ models with random production having Pareto decay with various demand patterns. The Exponential distribution includes increasing, decreasing and constant rate of production This project is divided in to four chapters. The chapter wise summary is given below.

In chapter I of the project a brief introduction regarding inventory models for deteriorating items is presented. The motivation of the present work is given in the focus of the project. A brief discussion about the Exponential and Pareto distribution are presented, since Exponential distribution is used to model the production and Pareto distributions is used to characterize the lifetime of commodity. The brief reviews about the EOQ models for deteriorating items and EPQ models for deteriorating items are presented in order to highlight the present work in its right perspective. From the brief review it is observed that much work has been reported in literature regarding economic production model for deteriorating items. Raafat (1991), Goyal and Giri (2001), Ruxian Li, et al. (2010), Pentico and Drake (2011) have reviewed the inventory models for deteriorating items. However, very little work has been reported in literature regarding EPQ models for deteriorating items with random production even through these models are much common in several production/manufacturing processes. The organization of the thesis is also presented.

In chapter II, an economic production quantity model for deteriorating items is developed and analyzed with the assumption that the replenishment is random and follows a Exponential distribution. It is also assumed that life time of commodity is random and follows a Pareto distribution. It is further assumed that demand is linear function of selling price. Here, it is assumed that the demand rate is of the form $f(s) = a - bs$, where 'a' and 'b'are the demand parameters and's' is the selling price. For different values of the demand parameters, the demand pattern includes constant, increasing and decreasing rates of demand. Assuming that shortages are allowed and fully backlogged, using the differential equations, the instantaneous state of inventory, the stock loss due to deterioration, the backlogged demand and maximum stock and shortages level are obtained. With suitable cost considerations the total profit function is derived. By maximizing the total profit the optimal ordering quantity, optimal selling price, optimal replenishment down time and optimal replenishment uptime are obtained. This model is extended to the case of without shortages. For both the models a numerical illustration from a food processing industry is presented.

It is observed that, for both models the replenishment (production) down time and total profit are decreasing when the cost per unit 'C', the replenishment (production rate) parameter 'λ' are increasing. As the unit holding cost 'C_1' increases, the optimal ordering quantity, total profit are increasing, the optimal selling price decreases. When the demand parameter 'a' increases, the total profit decreases. It is observed that, for the model with shortages, as the deteriorating parameters 'α', increases, the optimal replenishment (production) downtime, the optimal replenishment (production) uptime, the optimal selling price and the optimal ordering quantity are increasing, the total profit decreases. It is also observed that, for both models the demand parameter 'b' increases, the total profit increases.

From the sensitivity analysis of the model it is observed that, the deterioration parameter 'α' significantly influences the optimal replenishment down time and optimal ordering quantity. In both models the replenishment parameter 'λ' tremendously influence the total profit. It is also observed that in both models the total profit is highly sensitive to the replenishment parameters and moderately sensitive to the cost per unit time.

In chapter III, another economic production quantity model for deteriorating items is developed and analyzed with the assumption that the replenishment is random and follows a Exponential distribution. It is also assumed that life time of commodity is random and follows a Pareto distribution. Here, it is assumed that demand is a power function of time. It is further assumed that the demand rate is of the form$\lambda(t) = \frac{d.t^{\frac{1}{n}-1}}{n.T^{\frac{1}{n}}}$, where‘d’ is the total demand in the cycle, ‘n’ is indexing parameter and ‘T’ is the cycle length. For different values of parameter ‘n’, the demand rate may be increasing/decreasing/constant.

Assuming that shortages are allowed and fully backlogged and using the differential equations, the instantaneous state of inventory, the stock loss due to deterioration and the backlogged demand are derived. By minimizing the total cost function, the optimal replenishment down time, the optimal replenishment uptime and the optimal ordering quantity are obtained. This model is extended to the case of without shortages. For both models, results are illustrated with numerical examples from a cement producing company is presented. The sensitivity of the model with respect to costs and parameters is also studied.

It is observed that, in both the models, as deterioration parameter ‘α’ increases, the optimal replenishment (production) down time and the optimal ordering quantity are increasing. It is also observed that, in both models, when cost per unit ‘C’ and replenishment (production rate) parameter ‘λ’ are increasing, the optimal replenishment (production) down time and total cost per unit time are increasing. The increase in the replenishment (production rate) parameter ‘α’ increases the optimal replenishment (production) down time, the optimal ordering quantity and total cost per unit time. It is further observed that, in both models, when holding cost per unit ‘C_1’ and demand parameter ‘a’ are increasing, the total cost per unit time decreases. As deteriorating parameter ‘α’ and indexing parameter ‘n’ are increasing, the optimal replenishment (production) down time decreases.

It is further observed from the sensitivity analysis that, the replenishment (production rate) parameter ‘λ’ tremendously influence the optimal ordering quantity and total cost per unit time. In both models the demand parameters have significant influence on the optimal ordering quantity and total cost per unit time. It is also observed that the optimal replenishment downtime and the optimal ordering quantity are sensitive to unit costs ‘C’ and ‘h’.

In chapter IV, the results derived in the earlier chapters are summarized with conclusions. The scope for further work in this area of research is also pointed.

SCOPE FOR FURTHER RESEARCH:

In this project economic production quantity models are developed and analyzed for a single commodity under consideration. It is possible to develop EPQ models for multiple commodities using random production (variable rate of production). Throughout the project it is assumed that the money value remain constant over the period of time i.e. the inflation has no influence on the models. It is also possible to develop and analyze the EPQ models developed in this project with inflation (time values of money) which require further investigation.

In this project the production level inventory models for deteriorating items are developed with the assumption that the production is governed by laws of chance and the variable production can be characterized with Exponential distribution. It is also considered that the lifetime of commodity is random and follows Pareto distribution. Different demand patterns are considered to provide spectra of EPQ models. These EPQ models are having practical utilization in scheduling the production processes dealing with perishable commodities like sea food's jelly jams, cement, edible oil, agricultural products, chemicals and pharmaceuticals etc.

The lifetime distribution of the models also includes the distributions such as uniform as limiting cases. The Exponential rate of production can include increasing, decreasing and variable rates of production which provide a flexibility to implement the model for different situations. The production/operational managers can estimate the production parameters and deteriorating parameters from the historical date available in the records. The estimation of the cost can also be inferred from records of marketing and stores. These models include several of the earlier models as particular cases for limiting or specific values of the parameters.

It is highly possible to develop many more economic production quantity models with plausible conditions in order to utilize the resources more optimally, efficiently and effectively.

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INVENTORY MODEL of DECLINING GOODS with EXPONENTIAL REPLENISHMENT and DECLINE of PARETO HAVING SELLING PRICE-DEPENDENT DEMAND

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Abstract:

Stock models play a great role in determining optimum ordering policies and pricing policies of industrial and business situations. Much research has been published on the stock model for deteriorating products with finite and infinite replenishment. Nevertheless, very little work has been done on the stock model of degradation with random replenishment, which is a very useful situation, such as production, storage, environmental conditions, availability of raw materials, etc. It is therefore very necessary to develop an inventory model with a random replenishment. In this paper, designing and evaluating a model for deteriorating products, assuming that the reprocessing is random, follows the Exponential Distribution and further assuming that the commodity's lifespan is random, follows Pareto's distribution and demand is a selling price feature. The instantaneous condition of stock is derived from differential equations. The total cost shall be calculated on the basis of acceptable price factors. Through maximizing the overall profit function, the optimum ordering policies and pricing policies are obtained and the model is also displayed numerically using the parameters and the sensitivity analysis is carried out. The model sensitivity test has a significant impact on the ordering policies and pricing policies of the model. From The model random replenishment sensitivity analysis has a significant influence on the model's ordering policies and pricing policies. The model also uses some of the earlier models as specific cases for parameter specific values.

Keywords: Exponential density, random replenishment, selling price dependent demand and EPQ model, Pareto decay.

1) INTRODUCTION:

Inventory models generate more interest due to their ready use at different locations such as transport systems, production processes, warehouses and market yards, etc., A number of models of inventory were developed and analyzed in order to study different stock structures. The essence of the product, demand and replenishment are very important factors that have an effect on the supply systems. In many inventories systems assumed the replenishment is limitless, and these models of inventory are instantaneous, in addition that replenishment rate is fixed and finite.

The authors (3) analyzed the finite rate of production. The authors (1) developed two models; production is considered to be a function of the stock level in one system and output is a function of the demand rate in another. The author (2) studied the EOQ model was tested with no backorders. The authors (7) studied a stochastic stock model of two discrete systems of output. The authors (9) Analyzed stock order level model with adjustable deterioration frequency and alternative rate of replenishment.

The authors (10, 11) developed inventory models assuming the output is random and follows the distribution of Weibull. They assumed that the commodity's lifespan is random and follows an exponential distribution with a constant deterioration frequency. The authors (14) considered inventory model assuming replenishment is random and follows the distribution of Weibull and commodity lifetime is random and follows generalized distribution of Pareto with variable deterioration rate. The authors (10) inventory model for random replenishment intervals and imperfect quality items with fluctuations in demand. They assumed that replenishment is random and the commodity life is random with constant rate of deterioration.

Furthermore, in many other practical situations, such as the food processing industry, product life varies depending on selling price. It's also known that the commodity's lifespan has a finite upper limit and the rate of decay is proportionate to the time.. This nature could well be characterized by the distribution of Pareto. This nature could well be characterized by the distribution of Pareto. The distribution Pareto is capable of representing the lifetime of a commodity with a variable rate of decay. The pattern of demand is another important factor in the inventory system. Generally, demand is to be considered constant. Nevertheless, in some other production units dealing with food processing, mining, cement production and freight handling, demand depends on the selling price. The literature on stock models with selling price dependent has gathered further data.

The authors (5, 6, 8, 13-16) studied inventory models with price-dependent demand sales. The authors (4) have studied inventory models with selling price dependent demand and three parameter Weibull decay having stock dependent production. The authors (15) Studied stock models of price-dependent demand for sales generalized Pareto decay having random production.

Very little literature work has been done on randomly replenished stock models and Pareto decay with price dependent demand sales, which are very useful for obtaining optimal production schedules and ordering policies. Thus, this paper develops and analyzes an inventory model for the deterioration of items on the assumption that replenishment is random and follows an Exponential Distribution. The lifetime of the product is also assumed to be random and follows the distribution of Pareto and demand is assumed to be a linear function of the selling price. Assuming shortages are permitted and the instantaneous state of stock is completely backlogged. Use differential equations, the overall cost function and the rate of profit function are achieved. The optimal production schedule and optimal output quantity are obtained by optimizing the profit rate function. The sensitivity analysis is carried out by statistical comparison. The layout is generalized in the case of no shortages.

Assumptions of the model:

(i) The demand rate is depends on unit selling price which is $f(s) = a - b\,s$, $a>0$, $b<a$.

(ii)The replenishment is finite and fits the density function of the Exponential distribution $f(t) = \lambda e^{-\lambda t}, t > 0, \lambda > 0$

Therefore the instantaneous replenishment is $k(t) = \frac{f(t)}{1-F(t)} = \lambda,\ \lambda > 0$

(iii) Time of lead is zero

(iv) The length of the cycle T is fixed and known

(v) The shortages are permitted and completely backlogged

(vi) The deterioration unit has been lost

(vii) Instantaneous rate of deterioration is $h(t) = \frac{\alpha}{t}, t > 0$.

Notations of the model:

The use the following notations to the development of the model

Q: Order quantity in a single cycle

A: Ordering cost

C: Cost per unit

C_1: The cost of the inventory per unit of time

C_2: The cost of shortage per unit time

s: Price for selling per unit

F(s): Demand rate.

2) WITH SHORTAGES OF INVENTORY MODEL:

Consider the system of inventories where the inventory rate is at time t=0 is zero. Inventory levels increase over the time (0, t_1) due to additional demand replenishment and deterioration is met. When the inventory rate exceeds S, the replenishment ceases at time t_1. The stock is gradually decreasing due to demand and interval degradation (t_1, t_2). At t_2 the stock must generate null and back orders over the duration (t_2, t_3). At time t_3, after meeting the demand, the replenishment begins again and fulfills the backlog. The back orders are met during (t_3,T) and the stock level at the close of round T hits zero. The diagram schematic showing the instantaneous state of stock is shown in Figure 1

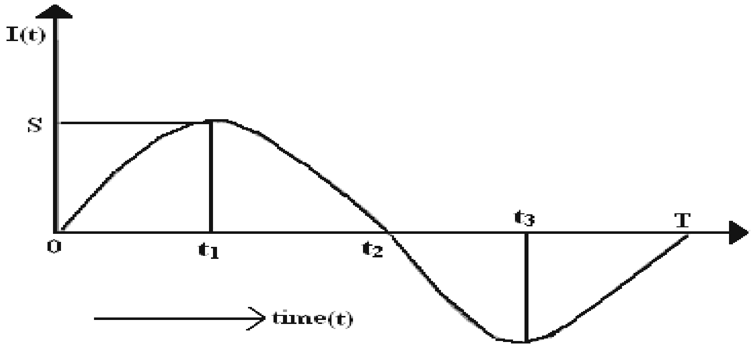


Fig 1: Diagram of schematics representing the inventory level.

Let I(t) be the system's inventory at ' t ' time (0≤t≤T). Differential equations governing the instant I(t) status over the T phase duration.

$$\frac{d}{dt}I(t) + h(t)I(t) = \lambda - (a - bs), 0 \le t \le t_1 \tag{1}$$

$$\frac{d}{dt}I(t) + h(t)I(t) = -(a - bs), t_1 \le t \le t_2 \tag{2}$$

$$\frac{d}{dt}I(t) = -(a - bs), t_2 \le t \le t_3 \tag{3}$$

$$\frac{d}{dt}I(t) = \lambda - (a - bs), t_3 \le t \le T \tag{4}$$

where, h(t) is as mentioned in equation (3),under the initial conditions I(0) = 0, I(t_1) = S, I(t_2) = 0 and I(T) = 0.

Replace h (t) in the equation (3) in equations (1) and (2) and solving differential

equations, the stock on hand shall be acquired as

$$I(t) = \frac{\lambda-(a-bs)}{\alpha+1}\left(\frac{t^{\alpha+1}-t_1^{\alpha+1}}{t^\alpha}\right) + S\left(\frac{t_1}{t}\right)^\alpha, 0 \le t \le t_1 \tag{5}$$

$$I(t) = \frac{-(a-bs)}{\alpha+1}\left(\frac{t^{\alpha+1}-t_1^{\alpha+1}}{t^\alpha}\right) + S\left(\frac{t_1}{t}\right)^\alpha, t_1 \le t \le t_2 \tag{6}$$

$$I(t) = (a - bs)(t_2 - t), t_2 \le t \le t_3 \tag{7}$$

$$I(t) = (\lambda - (a - bs))(t - T),\ t_3 \le t \le T \tag{8}$$

Loss of stock due to deterioration of the range (0, t)

$$L(t) = \int_0^t k(t)dt - \int_0^t (a - bs)dt - I(t)\ ; 0 \le t \le t_2$$

This implies

$$L(t) = \begin{cases} \lambda t - (a - bs)t - \frac{\lambda-(a-bs)}{\alpha+1}\left(\frac{t^{\alpha+1}-t_1^{\alpha+1}}{t^\alpha}\right) - S\left(\frac{t_1}{t}\right)^\alpha; 0 \le t \le t_1 \\ \lambda t_1 - (a - bs)t_1 + \frac{(a-bs)}{\alpha+1}\left(\frac{t^{\alpha+1}-t_1^{\alpha+1}}{t^\alpha}\right) - S\left(\frac{t_1}{t}\right)^\alpha;\ \ t_1 \le t \le t_2 \end{cases}$$

Loss of stock due to deterioration of the T-length cycle

$$L(T) = \lambda t_1 - (a - bs)t_2$$

The order quantity Q for the length cycle T is

$$Q = \int_0^{t_1} k(t)dt + \int_{t_3}^{T} k(t)dt = \lambda(\ t_1 + T - t_3) \tag{9}$$

From equation (5) and to use the initial conditions I (0) = 0, we get the value of 'S '

$$S = \frac{\lambda-(a-bs)}{\alpha+1} t_1 \tag{10}$$

From equation (6) and using the initial condition I (t_2) = 0, one can get

$$t_2 = \left(t_1^{\alpha+1} + \frac{S t_1^{\alpha}(\alpha+1)}{(a-bs)} \right) \tag{11}$$

On simplification

$$t_2 = \left(\frac{\lambda t_1^{\alpha+1}}{a-bs} \right)^{\frac{1}{\alpha+1}} \tag{12}$$

Let $K(t_1, t_2, t_3, s)$ be the total cost per time unit. Since the total cost is the amount of the cost collection, the cost of the items, the cost of keeping the stock, the total cost per unit time is.

$$K(t_1, t_2, t_3, s) = \frac{A}{T} + \frac{CQ}{T} + \frac{h}{T}\left(\int_0^{t_1} I(t)dt + \int_{t_1}^{t_2} I(t)dt \right) + \frac{\pi}{T}\left(\int_{t_2}^{t_3} -I(t)dt + \int_{t_3}^{T} -I(t)dt \right) \tag{13}$$

Substitute the values of I(t) and Q in the equation (13) one could obtain $K(t_1, t_2, t_3, s)$ as

$$K(t_1, t_2, t_3, s) = \frac{A}{T} + \frac{C}{T}\lambda(t_1 + T - t_3) + \frac{C_1}{T}\left[\int_0^{t_1} \left(\frac{\lambda-(a-bs)}{\alpha+1} \left(\frac{t^{\alpha+1} - t_1^{\alpha+1}}{t^{\alpha}} \right) + S\left(\frac{t_1}{t} \right)^{\alpha} \right) dt \right.$$

$$\left. + \int_{t_1}^{t_2} \left(\frac{-(a-bs)}{\alpha+1} \left(\frac{t^{\alpha+1} - t_1^{\alpha+1}}{t^{\alpha}} \right) + S\left(\frac{t_1}{t} \right)^{\alpha} \right) dt \right] + \frac{C_2}{T}\left[\int_{t_2}^{t_3} \big((a-bs)(t-t_2) \big) dt + \int_{t_3}^{T} \big((\lambda-(a-bs))(t-T) \big) dt \right] \tag{14}$$

On simplification

$$K(t_1, t_2, t_3, s) = \frac{A}{T} + \frac{C}{T}\lambda(t_1 + T - t_3) + \frac{C_1}{T}\left[\frac{\lambda}{\alpha+1}\left(\frac{t_1^2}{2} - \frac{t_1^2}{1-\alpha} \right) \right.$$

$$\left. - \frac{(a-bs)}{\alpha+1}\left(\frac{t_2^2}{2} - \frac{t_1^{\alpha+1} t_2^{-\alpha+1}}{1-\alpha} \right) + \frac{S t_2^{-\alpha+1} t_1^{\alpha}}{1-\alpha} \right] + \frac{C_2}{T}\left[(a-bs)\left(\frac{t_2^2}{2} - t_2 t_3 - \frac{T^2}{2} + T t_3 \right) + \lambda\left(\frac{T^2}{2} - T t_3 + \frac{t_3^2}{2} \right) \right] \tag{15}$$

Let $P(t_1, t_2, t_3, s)$ be the function of profit rate. Since the rate of profit is the total income per unit minus the total cost per unit time, we have.

$$P(t_1, t_2, t_3, s) = s\,(a-bs) - K(t_1, t_2, t_3, s) \tag{16}$$

Substituting the value of $K(t_1, t_2, t_3, s)$ in equation (16), one can get the profit rate function as

$$P(t_1, t_2, t_3, s) = s\,(a-bs) - \left[\frac{A}{T} + \frac{C}{T}\lambda(t_1 + T - t_3) + \frac{C_1}{T}\left[\frac{\lambda}{\alpha+1}\left(\frac{t_1^2}{2} - \frac{t_1^2}{1-\alpha} \right) \right. \right.$$

$$\left. \left. - \frac{(a-bs)}{\alpha+1}\left(\frac{t_2^2}{2} - \frac{t_1^{\alpha+1} t_2^{-\alpha+1}}{1-\alpha} \right) + \frac{S t_2^{-\alpha+1} t_1^{\alpha}}{1-\alpha} \right] + \frac{C_2}{T}\left[(a-bs)\left(\frac{t_2^2}{2} - t_2 t_3 - \frac{T^2}{2} + T t_3 \right) + \lambda\left(\frac{T^2}{2} - T t_3 + \frac{t_3^2}{2} \right) \right] \right] \tag{17}$$

Substituting equations (10) and (12) in equation (17), one can get the profit rate function in terms of ‘t_1’, ‘t_3’ and ’s’ as

$$P(t_1,t_3,s) = s\,(a-bs) - \frac{A}{T} - \frac{C}{T}\lambda(t_1+T-t_3) - \frac{C_1}{T}\left[\frac{\lambda}{\alpha+1}\left(\frac{t_1^2}{2}-\frac{t_1^2}{1-\alpha}\right)\right.$$

$$-\frac{(a-bs)}{\alpha+1}\left(\frac{1}{2}\left(\frac{\lambda t_1^{\alpha+1}}{a-bs}\right)^{\frac{2}{\alpha+1}} - \frac{t_1^{\alpha+1}}{1-\alpha}\left(\frac{\lambda t_1^{\alpha+1}}{a-bs}\right)^{\frac{-\alpha+1}{\alpha+1}}\right) + \left.\frac{t_1^{\alpha+1}}{1-\alpha}\left(\frac{\lambda-(a-bs)}{\alpha+1}\right)\left(\frac{\lambda t_1^{\alpha+1}}{a-bs}\right)^{\frac{-\alpha+1}{\alpha+1}}\right]$$

$$-\frac{C_2}{T}\left[(a-bs)\left(\frac{t_2^2}{2}-t_2t_3-\frac{T^2}{2}+Tt_3\right)+\lambda\left(\frac{T^2}{2}-Tt_3+\frac{t_3^2}{2}\right)\right]$$

On simplification

$$P(t_1,t_3,s) = s\,(a-bs) - \frac{A}{T} - \frac{C}{T}\lambda(t_1+T-t_3) - \frac{C_1}{T}\left[\frac{\lambda}{\alpha+1}\left(\frac{t_1^2}{2}-\frac{t_1^2}{1-\alpha}+\frac{t_1^2}{1-\alpha}\left(\frac{\lambda}{a-bs}\right)^{\frac{-\alpha+1}{\alpha+1}}\right)\right.$$

$$\left.-\frac{(a-bs)}{\alpha+1}\left(\frac{t_1^2}{2}\left(\frac{\lambda}{a-bs}\right)^{\frac{2}{\alpha+1}}\right)\right] - \frac{C_2}{T}\left[(a-bs)\left(\frac{t_2^2}{2}\left(\frac{\lambda}{a-bs}\right)-t_1\left(\frac{\lambda}{a-bs}\right)^{\frac{1}{\alpha+1}}t_3-\frac{T^2}{2}+Tt_3\right)+\lambda\left(\frac{T^2}{2}-Tt_3+\frac{t_3^2}{2}\right)\right] \qquad (18)$$

3) PRICING OPTIMAL AND POLICIES OF THE MODEL:

We get the optimum stock process policies under review in this section. We get the first order partial derivatives of $P(t_1,t_3,s)$ given in equation (18) with respect to t_1 and s and equal to zero to find optimal t_1 and s values. The maximization criterion of $P(t_1,t_3,s)$ is

$$D = \begin{bmatrix} \dfrac{\partial^2 P(t_1,t_3,s)}{\partial t_1^2} & \dfrac{\partial^2 P(t_1,t_3,s)}{\partial t_1,\partial t_3} & \dfrac{\partial^2 P(t_1,t_3,s)}{\partial t_1,\partial s} \\ \dfrac{\partial^2 P(t_1,t_3,s)}{\partial t_1,\partial t_3} & \dfrac{\partial^2 P(t_1,t_3,s)}{\partial t_3^2} & \dfrac{\partial^2 P(t_1,t_3,s)}{\partial t_3,\partial s} \\ \dfrac{\partial^2 P(t_1,t_3,s)}{\partial t_1,\partial s} & \dfrac{\partial^2 P(t_1,t_3,s)}{\partial t_3,\partial s} & \dfrac{\partial^2 P(t_1,t_3,s)}{\partial s^2} \end{bmatrix} < 0$$

Differentiating $P(t_1,t_3,s)$ given in equation (18) with respect to t_1 and zero, you can get

$$\frac{C\lambda}{T}+\frac{C_1}{T}\left[\frac{\lambda}{\alpha+1}\left(t_1-\frac{2t_1}{1-\alpha}+\frac{2t_1}{1-\alpha}\left(\frac{\lambda}{a-bs}\right)^{\frac{-\alpha+1}{\alpha+1}}\right)-\frac{(a-bs)}{\alpha+1}\left(t_1\left(\frac{\lambda}{a-bs}\right)^{\frac{2}{\alpha+1}}\right)\right]+\frac{C_2}{T}\left[(a-bs)\,t_1\left(\frac{\lambda}{a-bs}\right)-\left(\frac{\lambda}{a-bs}\right)^{\frac{1}{\alpha+1}}t_3\right] = 0 \qquad (19)$$

Differentiating $P(t_1,t_3,s)$ given in equation (18) with respect to t_3 and zero, you can get

$$\frac{C\lambda}{T}-\frac{C_2}{T}\left[\left(T-\left(\frac{\lambda}{a-bs}\right)^{\frac{1}{\alpha+1}}t_1\right)(a-bs)+\lambda(t_3-T)\right] = 0 \qquad (20)$$

Differentiating $P(t_1,t_3,s)$ given in equation (18) with respect to s and zero, you can get

$$(a-bs)-\frac{C_1}{T}\left[\left[\frac{\lambda}{(\alpha+1\,)}\frac{t_1^2}{(1-\alpha)}\lambda^{\frac{1-\alpha}{1+\alpha}}\left(\frac{\alpha-1}{\alpha+1}\right)(a-bs)^{\frac{-2}{\alpha+1}}-\frac{\lambda^{\frac{2}{1+\alpha}}}{2(\alpha+1)}t_1^2\left(\frac{\alpha-1}{\alpha+1}\right)(a-bs)^{\frac{-2}{\alpha+1}}\right]\right.$$

$$\left.-\frac{C_2}{T}\left[\frac{t_1^2}{2}\lambda^{\frac{2}{1+\alpha}}\left(\frac{\alpha-1}{\alpha+1}\right)(a-bs)^{\frac{-2}{\alpha+1}}-t_1t_3\lambda^{\frac{1}{1+\alpha}}\frac{\alpha}{\alpha+1}(a-bs)^{\frac{-1}{\alpha+1}}+b\left(\frac{T^2}{2}+Tt_3\right)\right]\right. = 0 \qquad (21)$$

Solving the equations (19), (20) and (21) at the same time, we obtain the optimum time at which replenishment t_1^* of t_1 is stopped and the optimal time t_3^* of t_3 at which replenishment is restarted after back-order accumulation and optimum selling price s^* of s is obtained.

The optimum quantity order Q * of Q in the T-length cycle is obtained by substituting the optimum values of t_1^*, t_3^* in the equation. (9) as

$$Q^* = \lambda(t_1^* + T - t_3^*) \tag{22}$$

4) NUMERICAL ILLUSTRATION:

We are addressing this section the model's solution by means of numerical illustration by inventory system replenishment (production) uptime, replenishment (production) downtime, optimum order quantity and total profit. It was assumed here that the commodity deteriorated and that shortages were permitted and completely backlogged. In order to demonstrate the model's solution procedure, the other parameter values and the model-related costs are as follows:

A = 2000, 2100, 2200, 2300; C = 8.5, 8.925, 9.35, 9.775,

C_1 = 20, 21, 22, 23; C_2= 0.05, 0.0525, 0.055, 0.0575,

α = 0.5, 0.525, 0.55, 0.575; λ= 0.9, 0.945, 0.99, 1.44,

a= 20, 21, 22, 23; b = 0.5, 0.525, 0.55, 0.575; T = 12 months.

Substitute optimal order quantity Q*, replenishment(production) uptime, replenishment (production)down time, optimal selling price, total profit are calculated and presented in Table 1.

Table 1 show that the deterioration and replenishment parameters have an enormous impact on optimum replenishment times, order size, selling price and total profit.

If cost of ordering ‘A’ increased from 2000 to 2300, then optimum downtime replenishment t_1^* increased from 4.68 to 6.3.8, optimum uptime replenishment t_3^* increased from 5.291 to 7.157, the optimal selling price s^* increases from 17.341 to 19.632.31, optimal quantity of order Q^* decreases from 10.059 to 9.481, the total profit P^* decreases from 40.252 to 34.473. The cost parameter C increased from 8 to 9.775, optimum down replenishment time increased from 4.468 to 4.915, optimum uptime replenishment decreases from 5.291 to 5.225, the optimal selling price decreases from 17.341 to 17.365, optimal quantity of order Q^* increased from 10.059 to 10.521 and the total profit increases from 40.252 to 42;.5.

Table 1
Optimum t_1 *, t_3 *, s *, Q * and P * values for various parameter values

A	**C**	**C_1**	**C_2**	**T**	**λ**	**α**	**a**	**b**	**t_1**	**t_3**	**s**	**Q**	**P**
2000	**8.5**	**20**	**0.05**	**12**	**0.5**	**0.9**	**20**	**0.5**	4.468	5.291	17.341	10.059	40.252
2100				**12**					4.963	6.166	18.157	9.718	37.377
2200				**12**					5.668	6.638	18.907	9.626	35.912
2300				**12**					6.358	7.157	19.632	9.481	34.473
	8.925			**12**					4.632	5.272	17.346	10.224	41.076
	9.350			**12**					4.780	5.250	17.354	10.377	41.824
	9.775			**12**					4.915	5.225	17.365	10.521	42.500
		21		**12**					4.334	5.321	17.341	9.912	40.129
		22		**12**					4.195	5.373	17.343	9.740	39.916
		23		**12**					4.055	5.443	17.347	9.550	39.632
			0.0525	**12**					4.436	5.133	17.340	10.172	40.033
			0.0550	**12**					4.396	4.966	17.340	10.287	39.773
			0.0575	**12**					4.351	4.798	17.341	10.398	39.486
				12	**0.525**				4.436	5.314	17.343	10.009	40.164
				12	**0.550**				4.404	5.337	17.345	9.960	40.072
				12	**0.575**				4.372	5.361	17.347	9.910	39.979
				12		**0.945**			4.397	5.426	17.372	10.368	40.116
				12		**0.990**			4.336	5.526	17.401	10.702	40.006
				12		**1.035**			4.335	5.527	17.402	10.710	40.004
				12			**21**		4.127	5.351	17.205	9.698	39.366
				12			**22**		3.751	6.784	23.001	8.071	38.710
				12			**23**		3.741	6.642	22.924	8.189	39.077
				12				**0.525**	5.142	5.682	17.869	10.315	38.337
				12				**0.550**	6.027	6.406	18.371	10.459	37.740
				12				**0.575**	6.842	7.201	18.804	10.477	36.296

As the cost of keeping stock 'C_1' increased from 20 to 23, then optimum downtime replenishment t_1^* decreases from 4.468 to 4.055, the uptime replenishment t_3^* increased from 5.291 to 5.443, the selling price increased from 17.341 to 17.347 optimal quantity of order decreases from 10.059 to 9.550 and the total profit decreases from 40.252 to 39.362. As the shortage cost 'C_2' increased from 0.05 to 0.0575, optimum downtime replenishment decreases from 4.468 to 4.351, optimum uptime replenishment decreases from 5.291 to 4.798, the optimal selling price decreases from 17.341 to 17.340, optimal quantity of order increased from 10.059 to 10.398 and total profit decreases from 40.252 to 39.486.

As replenishment parameter 'λ' increased from 0.5 to 0.575 units, optimum down time replenishment decreases from 4.468 to 4.372, optimum uptime replenishment increased from 5.291 to 5.361, the optimal selling price increases from 17.341 to 17.347, optimal quantity of order Q^* decreases from 10.059 to 9.91 and the total profit decreases from 40.252 to 39.979. As deteriorating parameter 'α' increased from 0.9 to 1.035, optimum down time replenishment increased from 4.252 to 4.335, optimum uptime replenishment increased from 5.291 to 5.527, the optimal selling price increased from 17.341 to 17.402, optimal quantity of order Q^* increases from 10.059 to 10.710, the total profit decreases from 40.252 to 40.004.

As demand parameter 'a' increased from 20 to 23 units, the optimum value of t_1^* decreases from 4.468 to 3.741, the optimum value of t_3^* increased from 5.291 to 6.642, the optimum value of s^* increased from 17.341 to 22.924, the optimum value of Q^* decreases from 10.059 to 8.189, the total profit P^* decreases from 40.252 to 39.077. Another demand parameter 'b' increased from 0.5 to 0.575, optimum down time replenishment increased from 4.486 to 6.842, optimum uptime replenishment increased from 5.291 to 7.201, the optimal selling price increases from 17.341 to 18.804, optimal quantity of order increased from 10.059 to 10.477, the total profit decreases from 40.252 to 36.296.

5) MODEL SENSITIVITY ANALYSIS:

The sensitivity analysis is carried out to analyze the effect on optimal policies of changes in process parameters and costs by varying the parameter at a time for the model being evaluated (-15%, -10%,-5%, 0%, 5%, 10%, 15%). The findings are shown in Table 2. Figure 2 shows the relationship between the optimum values and the parameters.

Table 2

System sensitivity analysis-with shortages

Parameters	**Optimum Policies**	**Change of parameters**						
		-15%	**-10%**	**-5%**	**0%**	**5%**	**10%**	**15%**
A	t_1^*	1.851	1.85	2.652	4.468	4.963	5.668	6.358
	t_3^*	3.991	3.996	4.94	5.291	6.166	6.638	7.157
	s^*	16.610	16.611	17.308	17.341	18.157	18.907	19.632
	Q^*	10.468	10.369	10.241	10.059	9.718	9.626	9.081
	P^*	42.749	42.649	41.053	40.252	37.377	35.912	34.473
C	t_1^*	4.078	4.077	4.284	4.468	4.632	4.78	4.915
	t_3^*	5.321	5.321	5.307	5.291	5.272	5.25	5.225
	s^*	17.341	17.341	17.339	17.341	17.346	17.354	17.365
	Q^*	9.681	9.68	9.88	10.059	10.224	10.377	10.521
	P^*	38.321	38.316	39.335	40.252	41.076	41.824	42.5
C_1	t_1^*	4.789	4.704	4.593	4.468	4.334	4.195	4.055
	t_3^*	5.424	5.331	5.292	5.291	5.321	5.373	5.443
	s^*	17.368	17.353	17.345	17.341	17.341	17.343	17.347
	Q^*	10.228	10.235	10.172	10.059	9.912	9.74	9.55
	P^*	40.442	40.306	40.257	40.252	40.129	39.916	39.632
C_2	t_1^*	4.532	4.515	4.494	4.468	4.436	4.396	4.351
	t_3^*	5.73	5.587	5.441	5.291	5.133	4.966	4.798
	s^*	17.349	17.346	17.343	17.341	17.34	17.34	17.341
	Q^*	9.721	9.835	9.947	10.059	10.172	10.287	10.398
	P^*	40.743	40.6	40.437	40.252	40.033	39.773	39.486
α	t_1^*	4.566	4.533	4.5	4.468	4.436	4.404	4.372
	t_3^*	5.228	5.248	5.269	5.291	5.314	5.337	5.361
	s^*	17.336	17.338	17.339	17.341	17.343	17.345	17.347
	Q^*	10.204	10.156	10.108	10.059	10.009	9.96	9.91
	P^*	40.495	40.418	40.337	40.252	40.164	40.072	39.979
λ	t_1^*	4.626	4.628	4.545	4.468	4.397	4.336	4.335
	t_3^*	4.863	4.857	5.104	5.291	5.426	5.526	5.527
	s^*	17.28	17.28	17.31	17.341	17.372	17.401	17.402
	Q^*	9.54	9.535	9.782	10.059	10.368	10.702	10.71
	P^*	40.532	40.536	40.395	40.252	40.116	40.006	40.004
a	t_1^*	8.041	7.888	6.102	4.468	4.127	3.751	3.745
	t_3^*	6.295	6.24	5.837	5.291	5.151	4.784	4.714
	s^*	20.495	20.441	18.977	17.341	17.205	16.001	15.963
	Q^*	10.572	10.484	10.138	10.059	9.698	8.071	8.128
	P^*	41.07	40.653	40.513	40.252	39.366	38.71	38.69
b	t_1^*	3.8	4.097	4.29	4.468	5.142	6.027	6.842
	t_3^*	6.586	5.87	5.569	5.291	5.682	6.406	7.201
	s^*	17.125	17.255	17.347	17.341	17.869	18.371	18.804
	Q^*	8.293	9.204	9.649	10.059	10.315	10.459	10.477
	P^*	40.786	40.478	40.183	40.252	38.337	37.74	36.296

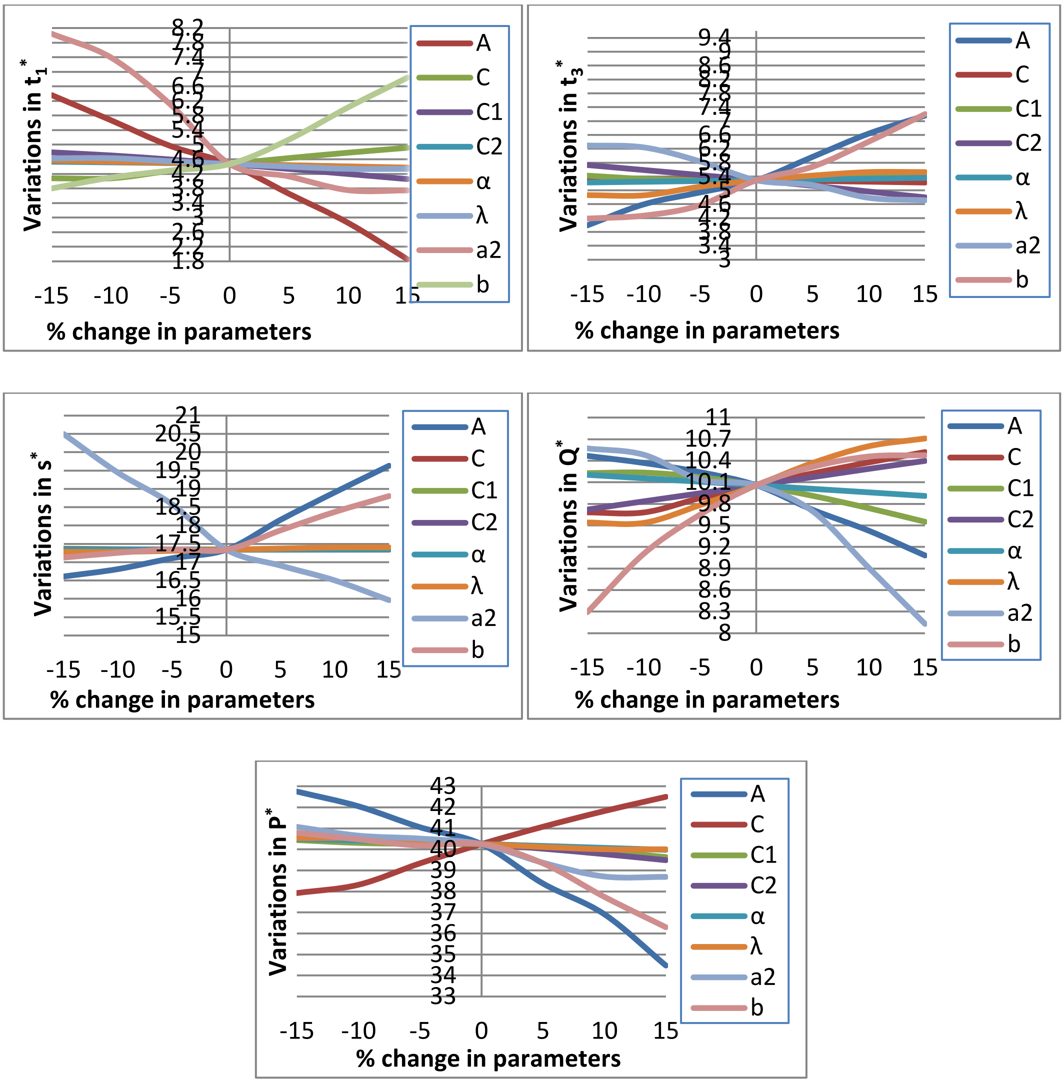


Fig 2 : Relationship between parameters and optimum shortage values

It is found that the costs affect the optimal order schedules of quantities and replenishment significantly. As the cost of ordering A decreases, optimum downtime replenishment t_1^*, the optimum uptime replenishment t_3^*, optimal selling price s^* are decreasing and optimum quantity of order Q^* and total profit P^* are increases. As ordering cost A increased, the optimum down time replenishment t_1^*, optimum uptime replenishment t_3^*, optimum selling price s^* are increases and optimum quantity of order Q^* and total profit $P^*$are decreasing. As t cost per unit C decreases, optimum uptime replenishment t_3^* and optimal selling price s^* are increases and optimum down time replenishment t_1^*, optimum quantity of order Q^* and total profit P^* are decreases. As cost per unit C increased, optimum uptime replenishment t_3^* and optimal selling price s^* are decreases and optimal down time replenishment t_1^*, optimum quantity of ordering Q^* and total profit P^* are increases.

The optimum values of t_1 *, t_3 *, s * Q*and P * are increasing as the holding cost ' C_1 ' decreases. The optimum values of t_1 *, t_3 *, s * Q*and P * are decreasing as the holding cost ' C_1 ' increases. As shortage cost 'C_2' decreases, optimum uptime replenishment t_3^*, the optimum selling price s^* and the total profit $P^*$are increases and optimal down time replenishment t_1^* and optimum quantity of order Q^* are decreasing. As shortage cost 'C_2' increases, the optimum uptime replenishment t_3^*, the optimum selling price s^* and the total profit $P^*$are decreasing and optimum down time replenishment t_1^* and optimum quantity of order Q^* are increases.

As replenishment parameter 'λ' decreases, the optimum values of t_1^*, Q^* and P^* are increases, the optimum value of t_3^*, s^* are decreases. As replenishment parameter 'λ' increases, the optimum values of t_1^*, Q^* and P^* are decreases and the optimum value of t_3^*, s^* are increases. The deteriorating parameter 'α' decreases, the optimum values of t_1^*, t_3^* $s^*$and P^* are increases and optimum ordering quantity Q^* is decrease. The deteriorating parameter 'α' increases, the optimum values of t_1^*, t_3^* $s^*$and P^* are decreases and optimum quantity of order Q^* is increase.

The demand parameter 'a' decreases, the optimum values of t_1^*, t_3^*, s^* $Q^*$and P^* are increases. The demand parameter 'a' increases, the optimum values of t_1^*, t_3^*, s^* $Q^*$and P^* are decreases. As another demand parameter 'b' decreases, the optimum values of t_3^*, P^* are increases and the optimum values of t_1^*, s^* and $Q^*$are decreases. As another demand parameter 'b' increases, the optimum values of t_3^*, P^* are decreases and the optimum values of t_1^*, s^* and $Q^*$are increases.

6) WITHOUT SHORTAGES OF INVENTORY MODEL:

In this section, the stock model is built and evaluated to deteriorate products without shortages. Here, shortages are considered not to be permitted and the inventory rate at time t= 0 is zero. During the period (0, t_1), the stock level increases due to excessive demand replenishment satisfaction and deteriorating. When the stock level reaches S, the replenishment stops at time t_1. The inventory is gradually decreasing due to demand and interval deterioration (t_1, T). The stock is zero at the time T. The diagram of schematic showing the instantaneous state of stock is shown in Figure 3

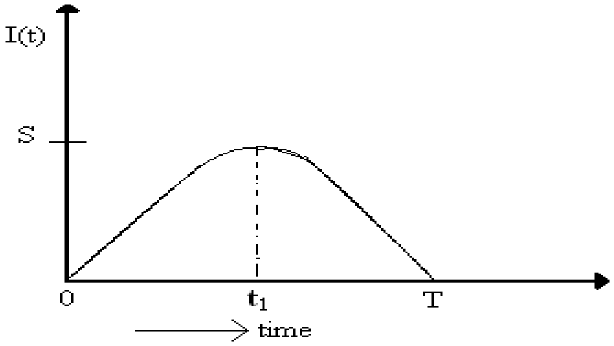


Fig 3: Diagram of schematics representing the inventory level.

Let I(t) be the inventory level of the system at ' t ' time (0≤t≤T). Differential equations that govern the instant state of I(t) over the duration of the T phase.

$$\frac{d}{dt}I(t) + h(t)I(t) = \lambda - (a - bs), 0 \le t \le t_1 \quad (23)$$

$$\frac{d}{dt}I(t) + h(t)I(t) = -(a - bs), t_1 \le t \le t_2 \quad (24)$$

where, h(t) is as mentioned in equation (3),under the initial conditions I(0) = 0, I(t_1) = S, I(t_2) = 0 and I(T) = 0.

Replace h(t) from equation (3) in equation (23) and (24) and solving differential equations, stock on hand shall be acquired as

$$I(t) = \frac{\lambda-(a-bs)}{\alpha+1}\left(\frac{t^{\alpha+1}-t_1^{\alpha+1}}{t^{\alpha}}\right) + S\left(\frac{t_1}{t}\right)^{\alpha}, 0 \le t \le t_1 \quad (25)$$

$$I(t) = \frac{-(a-bs)}{\alpha+1}\left(\frac{t^{\alpha+1}-t_1^{\alpha+1}}{t^{\alpha}}\right) + S\left(\frac{t_1}{t}\right)^{\alpha}, t_1 \leq t \leq t_2 \tag{26}$$

Loss of sotck due to deterioration of the range (0, t)

$$L(t) = \int_0^t k(t)dt - \int_0^t (a-bs)dt - I(t)\,, 0 \leq t \leq T$$

This implies

$$L(t) = \begin{cases} \lambda t - (a-bs)t - \frac{\lambda-(a-bs)}{\alpha+1}\left(\frac{t^{\alpha+1}-t_1^{\alpha+1}}{t^{\alpha}}\right) - S\left(\frac{t_1}{t}\right)^{\alpha}; 0 \leq t \leq t_1 \\ \lambda t_1 - (a-bs)t_1 + \frac{(a-bs)}{\alpha+1}\left(\frac{t^{\alpha+1}-t_1^{\alpha+1}}{t^{\alpha}}\right) - S\left(\frac{t_1}{t}\right)^{\alpha};\ \ t_1 \leq t \leq T \end{cases}$$

The order quantity Q for the length cycle T is

$$Q = \int_0^{t_1} k(t)dt \ = \lambda t_1 \tag{27}$$

From equation (25) and to use the initial conditions I (0) = 0, we get the value of 'S '

$$S = \frac{\lambda-(a-bs)}{\alpha+1} t_1 \tag{28}$$

Let the total cost per unit time be K(t_1 ,s). Since the total cost is the sum of the cost of set-up, the cost of units, the cost of holding inventory. Therefore the total cost is

$$K(t_1, s) = \frac{A}{T} + \frac{CQ}{T} + \frac{C_1}{T}\left(\int_0^{t_1} I(t)dt + \int_{t_1}^{T} I(t)dt\right) \tag{29}$$

Replacement of the value of I (t) and Q in the equation (25), (26) and (27) in equation (29), we obtain $K(t_1, s)$ as

$$K(t_1, s) = \frac{A}{T}\lambda t_1 + \frac{C_1}{T}\left\{\int_0^{t_1} \left[\frac{\lambda-(a-bs)}{\alpha+1}\left(\frac{t^{\alpha+1}-t_1^{\alpha+1}}{t^{\alpha}}\right) + S\left(\frac{t_1}{t}\right)^{\alpha} dt\right]\right.$$

$$\left. + \int_{t_1}^{T} \left[-\frac{(a-bs)}{\alpha+1}\left(\frac{t^{\alpha+1}-t_1^{\alpha+1}}{t^{\alpha}}\right) + S\left(\frac{t_1}{t}\right)^{\alpha} dt\right]\right\}$$

On simplification we get

$$K(t_1, s) = \frac{A}{T} + \frac{C}{T}\lambda t_1 + \frac{C_1}{T}\left[\frac{\lambda}{\alpha+1}\left(\frac{t_1^2}{2} - \frac{t_1^2}{1-\alpha}\right) - \frac{(a-bs)}{\alpha+1}\left(\frac{T^2}{2} - \frac{t_1^{\alpha+1}T^{1-\alpha}}{1-\alpha}\right) + \frac{S}{1-\alpha} t_1^{\alpha} T^{1-\alpha}\right] \tag{30}$$

Let P(t_1 ,s) be the function of the profit rate. Since the function of the profit rate is the total revenue per unit minus the total cost per unit time, we have

$$P(t_1, s) = s\,(a - bs) - K(t_1, s) \tag{31}$$

Substituting $K(t_1, s)$ value given in equation (30) in equation (31), one can get the rate of profit function as

$$P(t_1, s) = s\,(a - bs) - \frac{A}{T} - \frac{C}{T}\lambda t_1 - \frac{C_1}{T}\left[\frac{\lambda}{\alpha+1}\left(\frac{t_1^2}{2} - \frac{t_1^2}{1-\alpha}\right) - \frac{(a-bs)}{\alpha+1}\left(\frac{T^2}{2} - \frac{t_1^{\alpha+1}T^{1-\alpha}}{1-\alpha}\right) + \frac{S}{1-\alpha}t_1^{\alpha}T^{1-\alpha}\right] \tag{32}$$

Substituting ‘S’ value in equation (28) in equation (32), we get the profit rate function as

$$P(t_1, s) = s\,(a - bs) - \frac{A}{T} - \frac{C}{T}\lambda t_1 - \frac{C_1}{T}\left[\frac{\lambda}{\alpha+1}\left(\frac{t_1^2}{2} - \frac{t_1^2}{1-\alpha}\right) - \frac{(a-bs)}{\alpha+1}\left(\frac{T^2}{2} - \frac{t_1^{\alpha+1}T^{1-\alpha}}{1-\alpha}\right)\right.$$

$$\left. + \frac{\lambda - (a - bs)}{(\alpha + 1)(1 - \alpha)}t_1^{\alpha+1}T^{1-\alpha}\right]$$

On simplification we get

$$P(t_1, s) = s\,(a - bs) - \frac{A}{T} - \frac{C}{T}\lambda t_1 - \frac{C_1}{T}\left[\frac{\lambda}{\alpha+1}\left(\frac{t_1^2}{2} - \frac{t_1^2}{1-\alpha} + \frac{t_1^{\alpha+1}T^{1-\alpha}}{1-\alpha}\right) - \frac{(a-bs)}{2(\alpha+1)}T^2\right] \tag{33}$$

7) PRICING OPTIMAL AND THE MODEL POLICIES:

This section we get the optimal inventory system policies that are being studied. We equate the first order partial derivatives of P(t_1,s) with respect to t_1 in order to find the optimal values of t_1 and s and equate them to zero. The maximization requirement for P(t_1 ,s) is

$$D = \begin{vmatrix} \dfrac{\partial^2 P(t_1, s)}{\partial t_1^2} & \dfrac{\partial^2 P(t_1, s)}{\partial t_1 \partial s} \\ \dfrac{\partial^2 P(t_1, s)}{\partial t_1 \partial s} & \dfrac{\partial^2 P(t_1, s)}{\partial s^2} \end{vmatrix} < 0$$

Differentiate $P(t_1, s)$ with respect to t_1 and equating to zero, you can get

$$-\frac{C}{T}\lambda - \frac{C_1}{T}\left[\frac{\lambda}{\alpha+1}\left(t_1 - \frac{2t_1}{1-\alpha} + \frac{1+\alpha}{1-\alpha}t_1^{\alpha+1}T^{1-\alpha}\right)\right] = 0 \tag{34}$$

Differentiate $P(t_1, s)$ with respect to ‘s’ and equating to zero, you can get

$$-2bs - \frac{C_1}{T}\left(\frac{bT^2}{2(\alpha+1)}\right) = 0 \tag{35}$$

Through simultaneously resolving the equations (34) and (35), we obtain the optimum time at which the replenishment of t_1* of t_1 is to be stopped and the optimum unit selling price of s*. The optimum ordering Q* of Q in the length T cycle is obtained by replacing the optimal values of t_1 in (27) as.

$$Q^* = \lambda\, t_1 \tag{36}$$

8) NUMERICAL ILLUSTRATION:

In this section we analyze the model's solution method through a numerical example by obtaining the replenishment (production) time, optimum order size, and total profit. There, it believed that the product would deteriorate in nature and that shortages would not be tolerated and thoroughly reported away. In order to demonstrate the model's solution process, the values of the model's parameters and costs are:

A = 175, 183.75, 192.5, 203; C = 10, 10.5, 11, 11.5,

C_1 = 10, 9.5, 9, 8.5; λ= 5, 5.25, 5.5, 5.75, α= 0.5, 0.525, 0.55, 0.575

a= 25, 26.25, 27.5, 28.75; b = 0.5, 0.525, 0.55, 0.575; T = 12 months.

Replacing these values is calculated and presented in Table 3 the optimal order quantity Q^*, replenishment time, optimal selling price and optimal profit per unit time.

Table 3 shows that the parameters of deterioration and replenishment have a tremendous influence on the model's optimal values.

Table 3
Optimum t_1 *, s *, Q * and P * values for various parameter values

A	**C**	**C_1**	**T**	**A**	**λ**	**a**	**b**	**t_1**	**s**	**Q**	**P**
175.00	**10**	**10**	**12**	**0.5**	**5**	**25**	**0.5**	4.061	29.902	20.306	422.25
183.75			**12**					4.06	29.887	20.302	421.975
192.50			**12**					4.06	29.873	20.298	421.7
201.25			**12**					4.059	29.859	20.294	421.425
	10.5		**12**					4.056	29.897	20.281	422.163
	11.0		**12**					4.051	29.892	20.256	422.076
	11.5		**12**					4.046	29.887	20.231	421.988
		9.5	**12**					4.011	30.195	20.057	413.505
		9.0	**12**					4.001	30.397	20.005	402.298
		8.5	**12**					3.99	30.616	19.95	391.012
			12	**0.525**				4.086	30.061	20.429	420.922
			12	**0.550**				4.151	30.115	20.753	417.308
			12	**0.575**				4.495	28.775	22.474	413.804
			12		**5.25**			3.967	29.869	20.828	422.154
			12		**5.50**			3.874	29.864	21.309	422.067
			12		**5.75**			3.326	35.664	19.127	319.985
			12			**26.25**		4.19	31.934	20.948	443.006
			12			**27.50**		4.361	33.887	21.806	460.449
			12			**28.75**		5.946	31.135	29.728	365.349
			12				**0.525**	4.384	27.578	21.919	388.402
			12				**0.550**	5.654	21.568	28.27	288.672
			12				**0.575**	5.176	22.487	25.878	314.179

When ordering cost A increased from 175 to 201.25, the optimum replenishment time t_1^* decreases from 4.61 to 4.046, the optimum selling price s^* decreases from 29.902 to 29.887, the optimum quantity order Q^* decreases from 20.306 to 20.294, the total profit P^* decreases from 422.25 to 421.425. As cost per unit 'C' increased from 10 to 11.5, the optimum replenishment time t_1^* decreases from 4.61 to 4.59, the optimal selling price s^* increased from 29.902 to 34.85, the optimum quantity order Q^* decreases from 20.306 to 20.231, the total profit P^* decreases from 422.25 to 421.998. When the holding cost 'C_1' decreases from 10 to 8.5, the optimum replenishment time t_1^* decreases from 4.61 to 3.99 and the total profit P^* decreases from 422.25 to 391.012, the optimum selling price s^* increased from 29.902 to 30.616, the optimum quantity order Q^* decreases from 20.306 to 19.95.

As replenishment parameter 'λ' increased from 5 to 5.75 units, the optimum replenishment time decreases from 4.61 to 3.326, optimum selling price increased from 29.902 to 35.664 the optimum quantity order Q^* increased from 20.306 to 21.309 and the total profit decreases from 422.25 to 319.985. As deteriorating parameter 'α' increased from 0.5 to 0.575, the optimum replenishment time decreases from 4.61 to 4.495, the optimum selling price decreases from 29.902 to 28.775, the optimum quantity order Q^* increased from 20.306 to 22.474, the total profit decreases from 422.25 to 413.804.

The demand parameter 'a' increased from 25 to 28.75 units, the optimum value of t_1^* increased from 4.61 to 5.946, the optimum value of s^* increased from 29.902 to 31.135, the optimum value of Q^* increased from 20.902 to 29.728, the total profit P^* decreases from 422.25 to 365.349. Another demand parameter 'b' increased from 0.5 to 0.575, the optimum replenishment time increased from 4.61 to 5.176, the optimum selling price decreases from 29.902 to 22.487, the optimum quantity order increased from 20.902 to 25.878, the total profit decreases from 422.25 to 314.179.

9) MODEL SENSITIVITY ANALYSIS:

The sensitivity analysis is conducted to investigate the impact of changes in model parameters and costs on optimum policies by varying each parameter (-15%, -10%,-5%, 0%, 5%, 10%, 15%) at a time for the model being studied. Table 4 summarizes the findings. Figure 4 shows the relationship between the parameters and the replenishment schedule's optimum values.

Table 4

Analysis of model sensitivity - without shortages

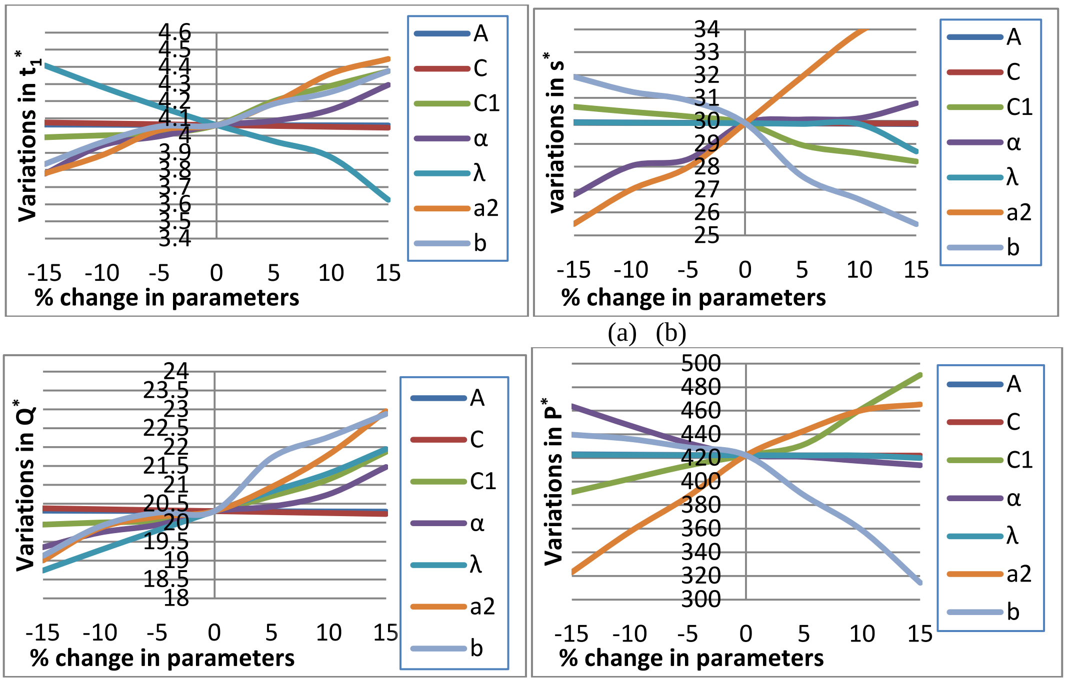
Parameters	**optimum policies**	**Change of parameters**						
		-15%	**-10%**	**-5%**	**0%**	**5%**	**10%**	**15%**
A	$\mathbf{t_1^*}$	4.064	4.063	4.062	4.061	4.06	4.06	4.059
	$\mathbf{s^*}$	29.944	29.93	29.916	29.902	29.887	29.873	29.859
	$\mathbf{Q^*}$	20.318	20.314	20.31	20.306	20.302	20.298	20.294
	$\mathbf{P^*}$	423.075	422.8	422.525	422.25	421.975	421.7	421.425
C	$\mathbf{t_1^*}$	4.076	4.071	4.066	4.061	4.056	4.051	4.046
	$\mathbf{s^*}$	29.916	29.911	29.907	29.902	29.897	29.892	29.887
	Q^*	20.382	20.357	20.331	20.306	20.281	20.256	20.231
	$\mathbf{P^*}$	422.507	422.422	422.336	422.25	422.163	422.076	421.988
$\mathbf{C_1}$	$\mathbf{t_1^*}$	3.99	4.001	4.011	4.061	4.201	4.29	4.375
	$\mathbf{s^*}$	30.616	30.397	30.195	29.902	28.227	28.585	28.941
	$\mathbf{Q^*}$	19.95	20.005	20.057	20.306	20.704	21.149	21.876
	$\mathbf{P^*}$	391.012	402.298	413.505	422.25	431.443	461.817	490.38
$\boldsymbol{\alpha}$	$\mathbf{t_1^*}$	3.778	3.942	3.996	4.061	4.086	4.151	4.295
	$\mathbf{s^*}$	26.767	28.031	28.325	29.902	30.061	30.115	30.775
	$\mathbf{Q^*}$	19.351	19.74	19.94	20.306	20.429	20.753	21.174
	$\mathbf{P^*}$	463.785	447.517	432.619	422.25	420.922	417.308	413.804
$\boldsymbol{\lambda}$	$\mathbf{t_1^*}$	4.408	4.283	4.168	4.061	3.967	3.874	3.626
	$\mathbf{s^*}$	29.924	29.915	29.908	29.902	29.869	29.864	28.664
	$\mathbf{Q^*}$	18.735	19.275	19.798	20.306	20.828	21.309	21.95
	$\mathbf{P^*}$	422.623	422.481	422.359	422.25	422.154	422.067	419.985
a	$\mathbf{t_1^*}$	3.78	3.886	4.031	4.061	4.19	4.361	4.446
	$\mathbf{s^*}$	25.495	26.992	27.975	29.902	31.934	33.887	35.135
	$\mathbf{Q^*}$	19.001	19.861	20.157	20.306	20.948	21.806	22.95
	$\mathbf{P^*}$	323.061	357.405	387.182	422.25	443.006	460.449	465.349
b	$\mathbf{t_1^*}$	3.834	3.961	4.053	4.061	4.184	4.254	4.376
	$\mathbf{s^*}$	31.929	29.078	31.894	29.902	27.578	26.568	25.487
	$\mathbf{Q^*}$	19.125	19.904	20.267	20.306	21.919	22.27	22.878
	$\mathbf{P^*}$	439.749	436.092	428.971	422.25	388.402	358.672	314.179

It is observed that the costs affect the optimum replenishment quantity and replenishment schedules significantly. As ordering cost A decreases, the optimum replenishment time t_1^*, the optimum quantity order Q^*, the optimum selling price s^* and total profit P^*increased. As ordering cost A increased, the optimum replenishment time t_1^*, the optimum quantity order Q^*, the optimum selling price s^* and total profit P^*decreases.

When the cost per unit C decreases, the optimum replenishment time t_1^*, the optimum quantity order Q^*, the optimum selling price s^* and total profit P^*increased. When the cost per unit C increased, the optimum replenishment time t_1^*, the optimum quantity order Q^*, the optimum and total profit P^*decreases. When the holding cost 'C_1' decreases, the optimum replenishment time t_1^*, the optimum quantity order Q^* are increased and total profit P^*, the optimum selling price s^* are decreases, when the holding cost 'C_1' increased, the optimum replenishment time t_1^*, the optimum quantity order Q^* and total profit P^* are increases and the optimum selling price s^* decreases.

As replenishment parameter 'α' decreases, the optimum values of s^* and P^* are decreases, the optimum replenishment time t_1^* and the optimum selling price s^* are increases. As replenishment parameter 'α' increased, the optimum values of s^* and P^* are increases, the optimum replenishment time t_1^* and the optimum selling price s^* are decreases. As deteriorating parameter 'λ' decreases, the optimum replenishment time t_1^*, the optimum selling price s^* and total profit P^* are increases and the optimum quantity order Q^* is decreases. As deteriorating parameter 'γ' increased, the optimum replenishment time t_1^*, the optimum selling price s^* and total profit P^* are decreases and the optimum quantity order Q^* is increased.

The demand parameter ‘a’ decreases, the optimum value of t_1^* is increased and the optimum values of s^* $Q^*$and P^* are decreasing. The demand parameter ‘a’ increased, the optimum value of t_1^* is decrease and the optimum values of s^* $Q^*$and P^* are increases. As another demand parameter ‘b’ decreases, the optimum values of t_1^*, s^* and Q^* are increases and the optimum value of P^* decreases. As another demand parameter ‘b’ increased, the optimum values of t_1^*, s^* and Q^* are decreasing and the optimum value of P^* increased.



(a) (b)

(c) (d)

Fig 4: Relationship between parameters and optimum values without shortages.

10) CONCLUSION:

A comparison of with and without shortages of this study, found that shortages were likely would increase total profit under Exponential recovery rate and price-dependent demand. Production or warehouse managers may obtain the optimum values of the production schedule or ordering policies by estimating parameters of demand, the production parameters and the deteriorating parameters in the past data. In this paper, an inventory model with random output and rate of Pareto decay was developed and analyzed. It is assumed that reprocessing is finite and follows Exponential distribution and further deterioration is also assumed to be random and follows the distribution of Pareto. Pareto decay describes delayed decay and uses Exponential and Uniform distributions as limiting cases. The model is useful for managers operating inventory control to make optimal decisions by estimating cost values from historical data. The proposed model is commonly used to examine circumstances that occur in areas such as factories, food and chemical processing industries where production is random and disaster-related. Some of the earlier models are also used by the model as specific cases for the specific parameter values.

Acknowledgement: This paper was developed under the UGC minor project No:MRP-7082/16 (SERO/UGC), I thank to UGC-INDIA and Director of Aditya Institute of Technology and Management, Tekkali for his continuous support for the project.

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Probabilistic stock model for exponentially replenished products and time-dependent Pareto decay

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Abstract:

This paper deals with stock model production when the rate of deterioration is random following Exponential distribution and product life is randomly accompanied by decline in Pareto. The demand rate is also assumed to be a time-dependent function here. Both cases were cared for in the development of the inventory model with shortage and without shortage. Shortages are completely backlogged whenever they are allowed. Through minimizing the overall cost function; the optimum ordering policies and pricing policies are obtained. Using numerical examples, findings are explained. The model's sensitivity analysis was performed to analyze the impact adjustments of the parameter values associated with the model.

Keywords: Exponential density, random replenishment, time dependent demand and EPQ model, Pareto decay.

I. INTRODUCTION:

Inventory models generate more interest due to their ready use at different locations such as transport systems, production processes, warehouses and market yards, etc., A number of models of inventory were developed and analyzed in order to study different stock structures. The essence of the product, demand and replenishment are very important factors that have an effect on the supply systems. In many inventories systems assumed the replenishment is infinite, and these models of inventory are instantaneous, in addition that replenishment rate is fixed and finite.

This paper contributes to the analysis and development of economic production quantity models for products that deteriorate with random production and decline in Pareto with time-dependent demand. The EPQ models are mathematical models which represent the inventory situation in a production or manufacturing system. The EPQ models can also be utilized for scheduling the optimal operating policies of market yards, warehouses, godowns, etc. In many of the inventory models the replenishment and production are considered synonymously. The economic production quantity models provide optimal decisions regarding the quantity to be produced (to be ordered) the production downtime and the production uptime. The EPQ models can be categorized into two categories namely, i) EPQ models for deteriorating items and ii) EPQ models for infinite lifetime. The EPQ models for deteriorating items gained lot of importance for the last two decades due to their ready applicability.

Much of the literature has been published EPQ models for the deterioration of items with various assumptions on demand, rate of deterioration and production. For developing inventory models characterization of the commodity's lifetime with a probability distribution is required. To ascribe the distribution of probability to the commodity's lifetime, one must consider the commodity's embedded lifetime process. The authors (2 and 3) assumed that an exponential distribution matches the lifetime of the product. The author (15) assumed distribution of gamma throughout the lifetime of the goods. The authors (8, 9 and 13) assumed distribution of Weibull throughout the lifetime of the products. The author (12) analyzed inventory models with generalized Pareto lifetime. The author (7) developed random-life stock models. But all these authors assumed that the replenishment rate is infinite and it is instantaneous.

Many others developed finite replenishment inventory models away from the infinite replenishment rate (production). The authors (4, 5 and 14) developed models of economic production with a constant rate of replenishment. The author (11) developed two different production rates were considered in one inventory system. The author (16) has developed stock models of production level with alternating replenishment rate. The author (1) has developed stock-dependent inventory models.

However, the rate of production is not constant or uniform in many manufacturing or production processes will have a variable rate of production. The production is to be considered as random due to various random factors such as transportation, raw materials, environment, skill levels, tool wear etc, are influencing the production process. This situation is evident in areas where the product is perishable, such as food processing industries, chemical factories, cement industries, etc. Very little work in the literature has been published regarding EPQ models With production (replenishment) at random except the models of the authors (10, 11) who have inventory models have been developed with random replenishment and constant deterioration rate and also the authors (6), who have developed model EPQ with random replenishment and deterioration rate variable. But in a lot of commodities the commodity's lifetime is random and has a minimum threshold period to start deterioration. Therefore, characterizing the commodity's lifetime with a Pareto distribution is reasonable. Hence, in this paper we create and analyze some randomly generated EPQ models with Pareto decay with demand pattern depending on time.

II. MODEL ASSUMPTIONS:

i) The Power of demand is the function of time which is $f(t) = \frac{dt^{1/n}}{nT^{1/n}}$ Where ' n ' is the parameter of indexing ' T ' is the length of the cycle and total demand is 'd '.

ii) The production (replenishment) is finite and fits the density function of the Exponential distribution

$$f(t) = \lambda e^{-\lambda t}, t > 0, \lambda > 0$$

Therefore the instantaneous replenishment is $k(t) = \frac{f(t)}{1-F(t)} = \lambda,\ \lambda > 0$

(iii) Leading period is zero

(iv) The length of the cycle T is fixed and known

(v) The shortages are allowable and completely backlogged

(vi) The deterioration unit has been lost

(vii) Instant deterioration rates is $h(t) = \frac{\alpha}{t}, t > 0$.

MODEL OBSERVATIONS:

The following observations are used for development of model

Q: Order the quantity in one cycle

A: Cost of ordering

C: Per unit cost

C_1: The cost of the inventory per unit of time

C_2: The cost of shortage per unit time

III. INVENTORY MODEL WITH SHORTAGES:

Consider the system of inventories where the inventory rate at time t=0 is zero. Inventory levels increase over the time (0, t_1) due to additional demand replenishment and deterioration is met. When the inventory rate exceeds S, the replenishment ceases at time t_1. The stock is gradually decreasing due to demand and interval degradation (t_1, t_2). At t_2 the stock must generate null and back orders over the duration (t_2, t_3). At time t_3, after meeting the demand, the replenishment begins again and fulfills the backlog. The back orders are met during (t_3,T) and the stock level at the close of round T hits zero. The diagram schematic showing the instantaneous state of stock is shown in Figure 1

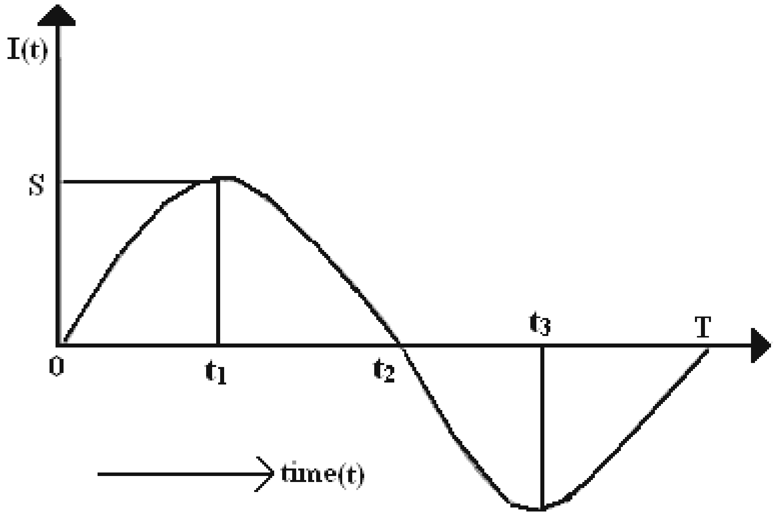


Fig 1: Inventory-level schematic diagram.

Let I(t) be the system's inventory at ' t ' time (0≤t≤T). Differential equations governing the instant I(t) status over the T phase duration.

$$\frac{d}{dt}I(t)+\frac{\alpha}{t}I(t)=\lambda-\frac{dt^{1/n}}{nT^{1/n}}, 0\le t\le t_1 \qquad (1)$$

$$\frac{d}{dt}I(t)+\frac{\alpha}{t}I(t)=-\frac{dt^{1/n}}{nT^{1/n}}, t_1\le t\le t_2 \qquad (2)$$

$$\frac{d}{dt}I(t)=-\frac{dt^{1/n}}{nT^{1/n}}, t_2\le t\le t_3 \qquad (3)$$

$$\frac{d}{dt}I(t) = \lambda - \frac{dt^{1/n}}{nT^{1/n}}, t_3 \le t \le T \qquad (4)$$

With the initial conditions I(0) = 0, I(t1) = S, I(t2) = 0 and I(T) = 0 and the differential equations are solved, the inventory on hand is obtained as ' t ' at the time.

$$I(t) = \frac{\lambda}{\alpha+1}\left(\frac{t^{\alpha+1}-t_1^{\alpha+1}}{t^{\alpha}}\right) + \frac{d}{T^{1/n}(1+\alpha n)}\left(\left(\frac{t_1}{t}\right)^{\alpha} t_1^{1/n} - t^{1/n}\right) + S\left(\frac{t_1}{t}\right)^{\alpha}, 0 \le t \le t_1 \qquad (5)$$

$$I(t) = \frac{-d}{T^{1/n}(1+\alpha n)}\left(\frac{t^{\alpha+1}-t_1^{\alpha+1}}{t^{\alpha}}\right) + S\left(\frac{t_1}{t}\right)^{\alpha}, t_1 \le t \le t_2 \qquad (6)$$

$$I(t) = \frac{d}{T^{1/n}}\left(t_2^{1/n} - t^{1/n}\right), t_2 \le t \le t_3 \qquad (7)$$

$$I(t) = \lambda(t-T) + \frac{d}{T^{1/n}}\left(T^{1/n} - t^{1/n}\right),\ t_3 \le t \le T \qquad (8)$$

Loss of stock due to deterioration of the range (0, t)

$$L(t) = \int_0^t k(t)dt - \int_0^t \mathrm{f(t)}dt - I(t), \quad 0 \le t \le t_2$$

$$L(t) = \begin{cases} \lambda t - \frac{d}{T^{1/n}} - \frac{\lambda}{\alpha+1}\left(\frac{t^{\alpha+1}-t_1^{\alpha+1}}{t^{\alpha}}\right) - \frac{d}{T^{1/n}}\left(\frac{t_1^{\alpha+1}-t^{\alpha+1}}{t^{\alpha}}\right) - S\left(\frac{t_1}{t}\right)^{\alpha}; 0 \le t \le t_1 \\ \lambda t_1 - \frac{d}{T^{1/n}} + \frac{d}{T^{1/n}(1+\alpha n)}\left(\frac{t^{\alpha+1}-t_1^{\alpha+1}}{t^{\alpha}}\right) + S\left(\frac{t_1}{t}\right)^{\alpha};\ t_1 \le t \le t_2 \end{cases}$$

Loss of stock due to deterioration of the T-length cycle

$$L(T) = \lambda t_1 - \mathrm{f(t)} t_2$$

The order quantity Q for the length cycle T is

$$Q = \int_0^{t_1} k(t)dt + \int_{t_3}^{T} k(t)dt$$

$$= \lambda(t_1 + T - t_3) \qquad (9)$$

From equation (5) and to use the initial conditions I (0) = 0, we get the value of 'S '

$$S = \frac{\lambda}{\alpha+1}t_1 - \frac{d}{T^{1/n}(1+\alpha n)}t_1^{1/n} \qquad (10)$$

when t = t_3, then equations (7) and (8) becomes

$$I(t) = \frac{d}{T^{1/n}}\left(t_2^{1/n} - t_3^{1/n}\right)$$

And $I(t) = \lambda(t_3 - T) + \frac{d}{T^{1/n}}\left(T^{1/n} - t_3^{1/n}\right)$

It is possible to compare the equations and simplify them

$$t_2 = T\left(\frac{\lambda}{d}(t_3 - T) + 1\right)^{n} \qquad (11)$$

Let $K(t_1, t_{2\,,} t_3)$ be the total cost. Since the total cost is the amount of the cost collection, the cost of the items, the cost of keeping the stock, the total cost is

$$K(t_1, t_{2,} t_3) = \frac{(A)}{T} + \frac{(CQ)}{T} + \frac{h}{T}\left(\int_0^{t_1} I(t)dt + \int_{t_1}^{t_2} I(t)dt\right) + \frac{\pi}{T}\left(\int_{t_2}^{t_3}(-I(t))dt + \int_{t_3}^{T}(-I(t))dt\right) \tag{12}$$

Replacing the values of I(t) and Q in equation (12) K(t_1,t_2, t_3) can be obtained as

$$K(t_1, t_2, t_3) = \frac{A}{T} + \frac{C}{T}\lambda(\, t_1 + T - t_3) + \frac{C_1}{T}\Bigg[\int_0^{t_1}\left(\frac{\lambda}{\alpha+1}\left(\frac{t^{\alpha+1} - t_1^{\alpha+1}}{t^{\alpha}}\right) + \frac{d}{T^{1/n}(1+\alpha n)}\left(\left(\frac{t_1}{t}\right)^{\alpha} t_1^{1/n} - t^{1/n}\right) + S\left(\frac{t_1}{t}\right)^{\alpha}\right) dt$$

$$+ \int_{t_1}^{t_2}\left(\frac{-d}{T^{1/n}(1+\alpha n)}\left(\frac{t^{\alpha+1} - t_1^{\alpha+1}}{t^{\alpha}}\right) + S\left(\frac{t_1}{t}\right)^{\alpha}\right) dt\Bigg]$$

$$+ \frac{C_2}{T}\left[\int_{t_2}^{t_3}\left(\frac{d}{T^{1/n}}\left(t_2^{1/n} - t^{1/n}\right)\right) dt + \int_{t_3}^{T}\left(\lambda(t - T) + \frac{d}{T^{1/n}}\left(T^{1/n} - t^{1/n}\right)\right) dt\right] \tag{13}$$

On integration and simplification one can get

$$K(t_1, t_2, t_3) = \frac{A}{T} + \frac{C}{T}\lambda(\, t_1 + T - t_3) + \frac{C_1}{T}\Bigg[\frac{\lambda}{\alpha+1}\left(\frac{t_1^2}{2} - \frac{t_1^2}{1-\alpha}\right) + \frac{d}{T^{1/n}(1+\alpha n)}\Bigg(\frac{t_1}{1-\alpha} + \frac{t_1^{\alpha+1/n} t_2^{1-\alpha}}{1-\alpha}$$

$$- \frac{t_1^{1+1/n}}{1-\alpha} - \frac{n}{n+1} t_2^{1+1/n}\Bigg) + \frac{S t_1^{\alpha} t_2^{1-\alpha}}{1-\alpha}\Bigg] + \frac{C_2}{T}\Bigg[\frac{d}{T^{1/n}}\left(\frac{1}{n+1}\left(t_2^{1+1/n} - T^{1+1/n}\right) - t_3 t_2^{1/n} + T^{1/n} t_3\right)$$

$$+ \lambda\left(\frac{T^2}{2} - T t_3 + \frac{t_3^2}{2}\right)\Bigg] \tag{14}$$

Replacing the value of 'S ' in equation (10) in the overall cost equation (14)

$$K(t_1, t_2, t_3) = \frac{A}{T} + \frac{C}{T}\lambda(\, t_1 + T - t_3) + \frac{C_1}{T}\Bigg[\frac{\lambda}{\alpha+1}\left(\frac{t_1^2}{2} - \frac{t_1^2}{1-\alpha} + \frac{t_1^{\alpha+1} t_2^{1-\alpha}}{1-\alpha}\right)$$

$$+ \frac{d}{T^{1/n}(1+\alpha n)}\left(\frac{t_1}{1-\alpha} - \frac{t_1^{\alpha+1/n}}{1-\alpha} - \frac{n}{n+1} t_2^{1+1/n}\right)\Bigg] + \frac{c_2}{T}\Bigg[\Bigg(\frac{d}{T^{1/n}}\frac{1}{n+1}\left(t_2^{1+1/n} - T^{1+1/n}\right)\Bigg)$$

$$- t_3 t_2^{1/n} + T^{1/n} t_3\Bigg) + \lambda\left(\frac{T^2}{2} - T t_3 + \frac{t_3^2}{2}\right)\Bigg] \tag{15}$$

We obtain a replace for the value of ' t_2 ' in equation (11) in the total cost formula (15) is

$$K(t_1, t_3) = \frac{A}{T} + \frac{C}{T}\lambda(\, t_1 + T - t_3) + \frac{C_1}{T}\Bigg[\frac{\lambda}{\alpha+1}\left(\frac{t_1^2}{2} - \frac{t_1^2}{1-\alpha} + \frac{t_1^{\alpha+1} T^{1-\alpha}}{1-\alpha}\left(\frac{\lambda}{d}(t_3 - T) + 1\right)^{n(1-\alpha)}\right)$$

$$+ \frac{d}{T^{1/n}(1+\alpha n)}\left(\frac{t_1}{1-\alpha} - \frac{t_1^{\alpha+1/n}}{1-\alpha} - \frac{n}{n+1} T^{1+1/n}\left(\frac{\lambda}{d}(t_3 - T) + 1\right)^{n+1}\right)\Bigg] + \frac{c_2}{T}\Bigg[d\left(\frac{T}{n+1}\left(\frac{\lambda}{d}(t_3 - T) + 1\right)^{n+1} - 1\right)$$

$$+ \lambda\left(\frac{T^2}{2} - \frac{t_3^2}{2}\right)\Bigg] \tag{16}$$

IV. OPTIMAL POLICIES AND PRICING OF THE MODEL:

We obtain the optimum stock process policies under review in this chapter. We obtain the first order partial derivatives of K(t_1,t_3) given in equation (16) with respect to t1 and t3 and compare them to zero in order to find the optimal values of t1 and t3. The K(t_1,t_3) minimization state is

$$D = \begin{vmatrix} \dfrac{\partial^2 K(t_1,t_3)}{\partial t_1^2} & \dfrac{\partial^2 K(t_1,t_3)}{\partial t_1 \partial t_3} \\ \dfrac{\partial^2 K(t_1,t_3)}{\partial t_3 \partial t_1} & \dfrac{\partial^2 K(t_1,t_3)}{\partial {t_3}^2} \end{vmatrix} > 0$$

Differentiating equation (16) to t_1 and to zero can be obtained

$$\frac{C\lambda}{T} + \frac{C_1}{T}\left[\frac{\lambda}{\alpha+1}\left(t_1 - \frac{2t_1}{1-\alpha} + \frac{\alpha+1}{1-\alpha} t_1^{\alpha} T^{1-\alpha}\left(\frac{\lambda}{d}(t_3 - T) + 1\right)^{n(1-\alpha)}\right) + \frac{d}{T^{1/n(1+\alpha n)}}\left(\frac{1}{1-\alpha} - \frac{n+1}{n(1-\alpha)} t_1^{1/n}\right)\right] = 0 \tag{17}$$

Differentiating equation (16) to t_3 and to zero can be obtained

$$\frac{C\lambda}{T} + \frac{C_1}{T}\left[t_1^{\alpha+1} T^{1-\alpha} n \frac{\lambda^2}{d(\alpha+1)}\left(\frac{\lambda}{d}(t_3 - T) + 1\right)^{n(1-\alpha)-1} - \frac{dnT^{1+1/n}}{T^{1/n(1+\alpha n)}}\frac{\lambda}{d}\left(\frac{\lambda}{d}(t_3 - T) + 1\right)^{n}\right]$$

$$+ \frac{c_2}{T}\left[\lambda T\left(\left(\frac{\lambda}{d}(t_3 - T) + 1\right) +^{n} 1\right) - t_3\right] = 0 \tag{18}$$

Simultaneously solving equations (17) and (18) we obtain the optimum time at which replenishment is to be stopped t_1* of t_1 and the optimum time t_1* of t_3 at which replenishment is to be restarted after backorders accumulation.

The quantity of ordering Q* of Q in the period cycle T is obtained by replacing the optimal values of t_1*, t_3* in equation (9) as

$$Q^* = \lambda(t_1^* + T - t_3^*) \tag{19}$$

V. NUMERICAL ILLUSTRATION OF THE MODEL:

In this section we address the model's solution method through a statistical example by obtaining a stock system's replenishment uptime, replenishment downtime, order quantity and total cost of an inventory. Here, it believed that the product would deteriorate in nature and that shortages would be allowed and fully logged back. The values of the parameters and costs associated with the model are used to illustrate the solution process of the model:

α = 0.5, 0.525, 0.55, 0.575, A = 1000, 1500, 1100, 1150; C = 10, 10.5, 11, 11.5,

C_1 = 20, 21, 22, 23; C_2= 0.5, 0.525, 0.55, 0.575, λ= 5, 5.25, 5.5, 5.75, n= 2, 2.1, 2.2, 2.3,

d = 80, 84, 88, 92; T = Twelve months..

Substitute these values optimum quantity of order, uptime of replenishment, downtime replenishment and total cost are calculated and presented in Table 1.

From Table 1 shows that that parameters of deterioration and replenishment have a tremendous influence on optimum replenishment times, order size, and total cost.

Table 1
Optimum values of t_1^*, t_3^*, Q^* and K^* for different values of parameters

A	**C**	**C_1**	**C_2**	**T**	**λ**	**α**	**n**	**d**	**t_1**	**t_3**	**Q**	**K**
1000	**10**	**20**	**0.5**	**12**	**5**	**0.5**	**2**	**80**	1.124	4.217	44.536	74.317
1050				**12**					1.178	4.296	44.412	76.177
1100				**12**					1.22	4.368	44.260	78.198
1150				**12**					1.257	4.439	44.089	80.278
	10.5			**12**					1.14	4.252	44.436	75.189
	11.0			**12**					1.166	4.291	44.374	75.899

	11.5			**12**					1.18	4.325	44.274	76.784
		21		**12**					1.122	4.175	44.736	72.313
		22		**12**					1.12	4.136	44.916	70.303
		23		**12**					1.117	4.101	45.078	68.288
			0.525	**12**					1.133	4.229	44.516	74.734
			0.55	**12**					1.141	4.242	44.495	75.15
			0.575	**12**					1.15	4.255	44.474	75.565
				12	**5.25**				1.25	4.43	46.305	79.757
				12	**5.50**				0.82	4.666	44.842	85.951
				12	**5.75**				1.191	4.926	47.524	89.099
				12		**0.525**			1.175	4.239	44.681	74.76
				12		**0.550**			1.215	4.252	44.814	75.279
				12		**0.575**			0.841	4.319	42.612	76.739
				12			**2.1**		1.185	4.297	44.437	76.737
				12			**2.2**		0.803	4.391	42.055	80.051
				12			**2.3**		1.274	4.442	44.165	81.401
				12				**84**	0.997	4.012	44.924	67.223
				12				**88**	0.884	3.808	45.38	59.387
				12				**92**	0.802	3.613	45.945	50.811

If cost of ordering ‘A’ increases from 1000 to 1150, then optimum quantity of order Q^* decreases from 44.536 to 44.089, optimum downtime replenishment t_1^* increases from 1.124 to 1.257, optimum uptime replenishment t_3^* increases from 4.217 to 4.439 and total cost K^*, increases from 74.317 to 80.278. The parameter of cost ‘C’ increases between 10 to 11.5 optimum quantity of order Q^* decreases between 44.536 to 44.274 optimum downtime replenishment t_1^* decreases from 1.124 to 1.18, optimum uptime replenishment t_3^* increases from 4.217 to 4.325, and total cost K^*, increases from 74.317 to 76.784.

When inventory holding cost ‘C_1’ increases from 20 to 23, optimum quantity of order Q^* increases from 44.536 to 45.078, optimum downtime replenishment t_1^* decreases from 1.124 to 1.117, optimum uptime replenishment t_3^* decreases from 4.217 to 4.101 and total cost K^*, decreases from 74.317 to 68.288. As shortage cost ‘C_2’ increases between 0.5 to 0.575, then optimum quantity of order Q^* decreases from 44.536 to 44.474, optimum downtime replenishment t_1^* decreases from 1.124 to 1.15, optimum uptime replenishment t_3^* increases from 4.217 to 4.255 and total cost K^*, increases from 74.317 to 89.099.

If replenishment parameter ‘λ’ increases between 5 to 5.75 then optimum quantity of order Q^* increases from 44.536 to 47.524, optimum downtime replenishment t_1^* increases from 1.124 to 1.191, optimum uptime replenishment t_3^* increases from 4.217 to 4.926 and total cost K^*, increases from 74.317 to 89.099. When deteriorating parameter ‘α’ increases between 0.5 to 0.575 optimum quantity of order Q^* decreases from 44.536 to 42.612, optimum downtime replenishment t_1^* decreases from 1.124 to 0.841, optimum uptime replenishment t_3^* increases from 4.217 to 4.319 and total cost K^*, increases from 74.317 to 76.739.

The parameter of indexing ‘n’ increases between 2 to 2.3, optimum quantity of order Q^* decreases from 44.536 to 44.165, optimum downtime replenishment t_1^* increases from 1.124 to 1.274, optimum uptime replenishment t_3^* increases from 4.217 to 4.442 and the total cost K^*, increases from 74.317 to 81.401. As parameter of demand ‘d’ increases from 80 to 92, optimum quantity of order Q^* increases from 44.536 to 45.945, optimum downtime replenishment t_1^* decreases from 1.124 to 0.802, optimum uptime replenishment t_3^* decreases from 4.217 to 3.613 and total cost K^*, decreases from 74.317 to 50.811.

VI. MODEL SENSITIVITY ANALYSIS:

The analysis of sensitivity is carried out to analyze the effect on optimal policies of changes in process parameters and costs by varying the parameter at a time for the model being evaluated (-15%, -10%,-5%, 0%, 5%, 10%, 15%). The findings are shown in Table 2. Figure 2 shows the relationship between the optimum values and the parameters.

It is found that the costs affect the optimal order schedules of quantities and replenishment significantly. As cost of ordering A decreases, the optimum downtime replenishment t_1^*, optimum uptime replenishment t_3^* and total cost K^* are decreases and optimum quantity of order Q^* increases. As cost of ordering A increases, the optimum downtime replenishment t_1^*, optimum uptime replenishment t_3^* and total cost K^* are increases and optimum quantity of order Q^* decreases. When cost per unit C decreases, optimum uptime replenishment t_3^*, optimum downtime replenishment t_1^* and total cost K^* are decreases and optimum quantity of order Q^* increases. When cost per unit C increases, optimum uptime replenishment t_3^*, optimal downtime replenishment t_1^* and total cost K^* are increases and optimum quantity of order Q^* decreases.

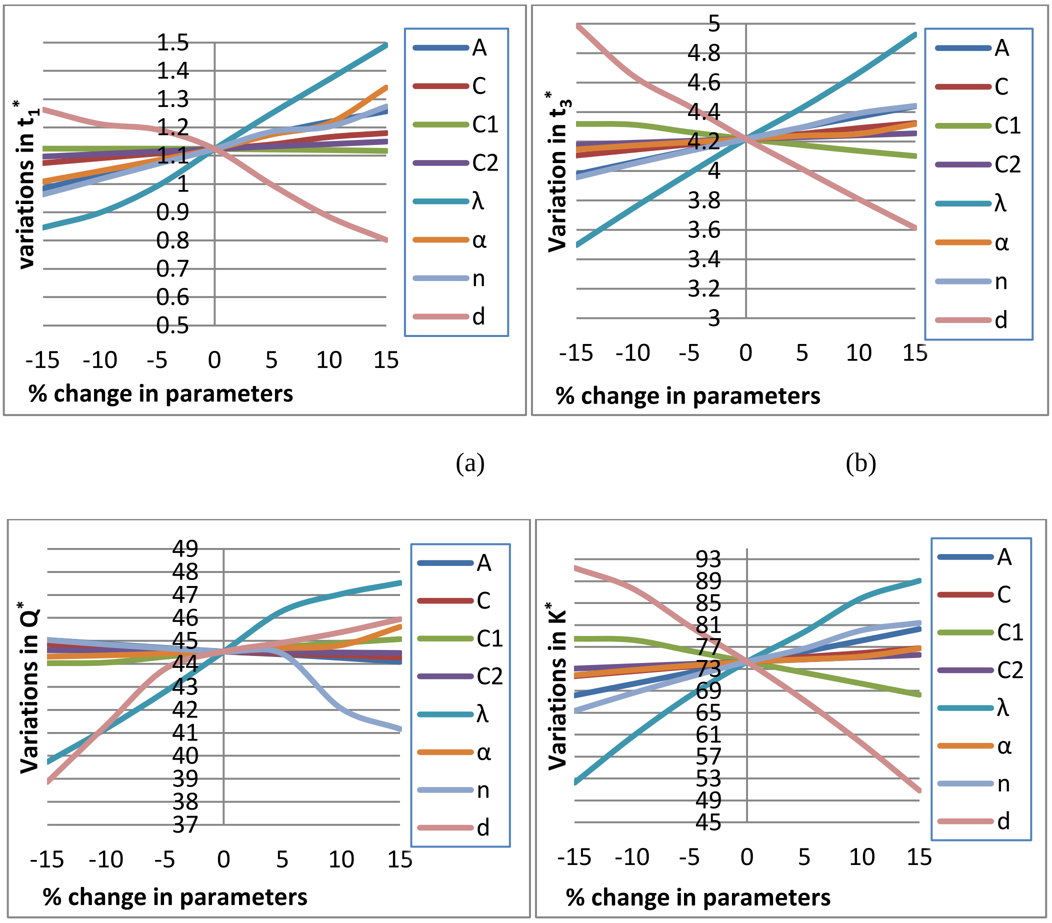
When holding cost 'C_1' decreases, optimum uptime replenishment t_3^*, the optimum downtime replenishment t_1^* and total cost K^* are increases and optimum quantity of order Q^* decreases. When holding cost 'C_1' increases, optimum uptime replenishment t_3^*, optimum downtime replenishment t_1^* and total cost K^* are decreases and optimum quantity of order Q^* increases. If shortage cost 'C_2' decreases, then optimum uptime replenishment t_3^*, optimal downtime replenishment t_1^* and total cost K^* are decreases and optimum quantity of order Q^* increases. If shortage cost 'C_2' increases, then optimum uptime replenishment t_3^*, optimum downtime replenishment t_1^* and total cost K^* are increases and optimal quantity of order Q^* decreases.

Table 2
System Sensitivity analysis - with shortages

Parameters	**optimum policies**	**Variations in parameters**						
		-15%	**-10%**	**-5%**	**0%**	**5%**	**10%**	**15%**
A	t_1^*	0.984	1.032	1.079	1.124	1.178	1.22	1.257
	t_3^*	3.977	4.059	4.139	4.217	4.296	4.368	4.439
	Q^*	45.038	44.865	44.698	44.536	44.412	44.26	44.089
	K^*	68.165	70.235	72.286	74.317	76.177	78.198	80.278
C	t_1^*	1.074	1.091	1.108	1.124	1.14	1.166	1.18
	t_3^*	4.105	4.143	4.18	4.217	4.252	4.291	4.325
	Q^*	44.846	44.741	44.637	44.536	44.436	44.374	44.274
	K^*	71.681	72.563	73.442	74.317	75.189	75.899	76.784
C_1	t_1^*	1.125	1.125	1.125	1.124	1.122	1.12	1.117
	t_3^*	4.319	4.314	4.263	4.217	4.175	4.136	4.101
	Q^*	44.031	44.058	44.311	44.536	44.736	44.916	45.078
	K^*	78.502	78.303	76.314	74.317	72.313	70.303	68.288
C_2	t_1^*	1.097	1.106	1.115	1.124	1.133	1.141	1.15
	t_3^*	4.178	4.191	4.204	4.217	4.229	4.242	4.255
	Q^*	44.595	44.576	44.556	44.536	44.516	44.495	44.474
	K^*	73.065	73.483	73.901	74.317	74.734	75.15	75.565
λ	t_1^*	0.846	0.899	0.993	1.124	1.25	1.37	1.491
	t_3^*	3.497	3.746	3.985	4.217	4.43	4.666	4.926
	Q^*	39.736	41.187	42.789	44.536	46.305	44.842	47.524
	K^*	52.24	60.592	68.045	74.317	79.757	85.951	89.099
α	t_1^*	1.009	1.046	1.084	1.124	1.175	1.215	1.341
	t_3^*	4.146	4.172	4.195	4.217	4.239	4.252	4.319
	Q^*	44.316	44.371	44.444	44.536	44.681	44.814	45.612
	K^*	71.866	72.779	73.598	74.317	74.76	75.279	76.739
n	t_1^*	0.963	1.017	1.071	1.124	1.185	1.203	1.274
	t_3^*	3.956	4.048	4.135	4.217	4.297	4.391	4.442

	Q^*	45.038	44.844	44.68	44.536	44.437	42.055	41.165
	K^*	65.358	68.554	71.541	74.317	76.737	80.051	81.401
d	**t_1^***	1.263	1.212	1.192	1.124	0.997	0.884	0.802
	t_3^*	4.991	4.647	4.437	4.217	4.012	3.808	3.613
	Q^*	40.863	40.325	43.778	44.536	44.924	45.38	45.945
	K^*	91.428	87.721	80.823	74.317	67.223	59.387	50.811

The replenishment parameter ‘λ’ decreases, optimum values t_3^*, t_1^*, $Q^*$and K^* are decreases. If replenishment parameter ‘λ’ increases then optimum values t_3^*, t_1^*, $Q^*$and K^* are increases. If parameter of deterioration ‘α’ decreases then optimum uptime replenishment t_3^* increases, optimum downtime replenishment t_1^*, optimum quantity of order Q^* and total cost K^* are decreases. As parameter of deterioration ‘α’ increases, optimum uptime replenishment t_3^* increases, optimum downtime replenishment t_1^*, optimum quantity of order Q^* and total cost K^* are increases.



(a) (b)

(c) (d)

Fig 2 : Relationship between parameters and optimum shortage values

If indexing parameter ‘n’ decreases, then optimum values t_3^*, t_1^*, $Q^*$and K^* are decreases. If indexing parameter ‘n’ increases, then optimum values t_3^*, t_1^*, $Q^*$and K^* are increases. When demand parameter ‘d’ decreases, optimum uptime replenishment t_3^*, optimum downtime replenishment t_1^* and total cost K^* are increases and optimum quantity of order Q^* decreases. When the demand parameter ‘d’ decreases, optimum uptime replenishment t_3^*, optimum downtime replenishment t_1^* and total cost K^* are increases and optimum quantity of order Q^* increases.

VII. INVENTORY MODEL WITHOUT SHORTAGES:

In this section, the stock model is built and evaluated to deteriorate products without shortages. Here, it presumed that shortages are not allowed and that inventory rate at time t = 0 is zero. During the time (0, t_1) the inventory rate rises due to excess replenishment after demand fulfilment and deterioration. When the inventory rate exceeds S, the replenishment ends at time t_1. The stock is gradually decreasing due to demand and interval deterioration (t_1, T). The stock hits zero at the time T. The diagram showing the instantaneous stock status is shown in Figure 3

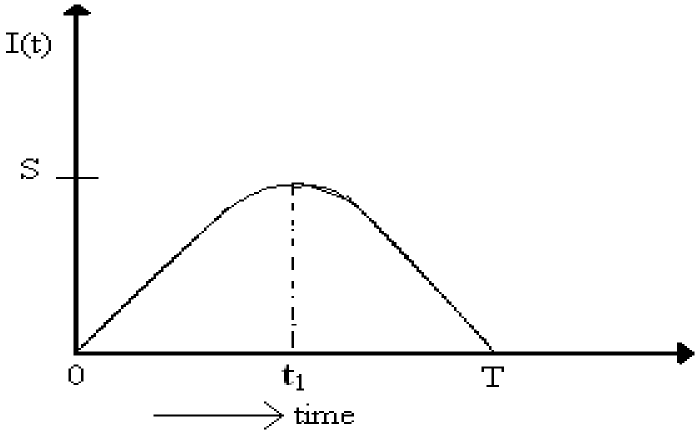


Fig 3: Schematic diagram showing the degree of the stocks.

Let I(t) be the inventory level of the system at ' t ' time (0≤t≤T). Differential equations that govern the instant state of I(t) over the duration of the T phase.

$$\frac{d}{dt}I(t)+\frac{\alpha}{t}I(t) = \lambda-\frac{d.t^{\frac{1}{n}-1}}{n.T^{\frac{1}{n}}}; \qquad 0\le t\le t_1 \tag{20}$$

$$\frac{d}{dt}I(t)+\frac{\alpha}{t}I(t) = -\frac{d.t^{\frac{1}{n}-1}}{n.T^{\frac{1}{n}}}; \qquad t_1\le t\le T \tag{21}$$

Using initial conditions, I(0) = 0, I(t_1) = S and I(T) = 0 and the differential equations are solved, the stock on hand is obtained as ' t ' at the time.

$$I(t)=\frac{\lambda}{\alpha+1}\left(t-\left(\frac{t_1}{t}\right)^{\alpha}t_1\right)+\frac{d}{T^{1/n}(1+\alpha n)}\left(\left(\frac{t_1}{t}\right)^{\alpha}{t_1}^{1/n}-t^{1/n}\right)+S\left(\frac{t_1}{t}\right)^{\alpha};0\le t\le t_1 \tag{22}$$

$$I(t)=\frac{d}{T^{1/n}(1+\alpha n)}\left(\left(\frac{t_1}{t}\right)^{\alpha}{t_1}^{1/n}-t^{1/n}\right)+S\left(\frac{t_1}{t}\right)^{\alpha};\ ;\ t_1\le t\le \mathrm{T} \tag{23}$$

Loss of stock due to interval deterioration (0, t)

$$L(t)=\int_0^t k(t)dt-\int_0^t \mathrm{f(t)}dt-I(t)\,,0\le t\le T$$

$$L(t)=\begin{cases}\lambda\mathrm{t}-\dfrac{dt^{1/n}}{T^{1/n}}\dfrac{\lambda}{\alpha+1}\left(t-\left(\dfrac{t_1}{t}\right)^{\alpha}t_1\right)+\dfrac{d}{T^{1/n}(1+\alpha n)}\left(\left(\dfrac{t_1}{t}\right)^{\alpha}{t_1}^{1/n}-t^{1/n}\right)+S\left(\dfrac{t_1}{t}\right)^{\alpha};\ 0\le t\le t_1\\ \lambda\mathrm{t}_1+\dfrac{d}{T^{1/n}(1+\alpha n)}\left(\left(\dfrac{t_1}{t}\right)^{\alpha}{t_1}^{1/n}-t^{1/n}\right)+S\left(\dfrac{t_1}{t}\right)^{\alpha};\qquad t_1\le t\le t_2\end{cases}$$

Ordering quantity Q for the length cycle T is

$$Q = \int_0^{t_1} k(t)dt = \lambda t_1 \tag{24}$$

Apply the initial condition I (0) = 0 from equation (22) we get the value of 'S ' as

$$S = \frac{\lambda}{\alpha+1} t_1 - \frac{d}{T^{1/n}(1+\alpha n)} {t_1}^{1/n} \tag{25}$$

Let K(t_1) be the total cost per time per unit. Because the total cost is the amount of the cost of set-up, the cost of items, the cost of keeping stock. The total cost of this is

$$K(t_1) = \frac{A}{T} + \frac{CQ}{T} + \frac{h}{T}\left(\int_0^{t_1} I(t)dt + \int_{t_1}^{T} I(t)dt\right) \tag{26}$$

We obtain K(t_1) as a substitute for the value of I (t) and Q given in equation (23), (24) and (25) as equation (26).

$$K(t_1) = \frac{A}{T} + \frac{C}{T}\lambda t_1$$

$$+\frac{C_1}{T}\left[\frac{\lambda}{\alpha+1}\int_0^{t_1}\left(t - \left(\frac{t_1}{t}\right)^{\alpha} t_1\right)dt + \frac{d}{T^{1/n}(1+\alpha n)}\int_0^{t_1}\left(\left(\frac{t_1}{t}\right)^{\alpha} {t_1}^{1/n} - t^{1/n}\right)dt + S\int_0^{t_1}\left(\frac{t_1}{t}\right)^{\alpha} dt\right.$$

$$\left.-\frac{d}{T^{1/n}(1+\alpha n)}\int_{t_1}^{T}\left(\left(\frac{t_1}{t}\right)^{\alpha} {t_1}^{1/n} - t^{1/n}\right)dt + S\int_{t_1}^{T}\left(\frac{t_1}{t}\right)^{\alpha} dt\right]$$

On integration and simplification one can get

$$K(t_1) = \frac{A}{T} + \frac{C}{T}\lambda t_1 + \frac{C_1}{T}\left[\frac{\lambda}{\alpha+1}\left(\frac{t_1^2}{2} - \frac{t_1^2}{1-\alpha}\right) + \frac{d}{T^{1/n}(1+\alpha n)}\left(\frac{t_1^{1+1/n}}{1-\alpha} - \frac{n t_1^{1+1/n}}{n+1}\right)\right.$$

$$\left.+\frac{S{t_1}^{\alpha}}{1-\alpha} - \frac{d}{T^{1/n}(1+\alpha n)}\left(\left(T^{1+1/n} - t_1^{1+1/n}\right)\left(\frac{n}{n+1} - \frac{1}{1-\alpha}\right)\right) + \frac{S{t_1}^{\alpha}}{1-\alpha}\left(T^{-\alpha+1} - t_1^{-\alpha+1}\right)\right] \tag{27}$$

Substituting the value of ‘S’ in given equation (3.7.6) in the cost equation (3.7.8), one can get

$$K(t_1) = \frac{A}{T} + \frac{C}{T}\lambda t_1 + \frac{C_1}{T}\left[\frac{\lambda}{\alpha+1}\left(\frac{t_1^2}{2} - \frac{2t_1^2}{1-\alpha} + \frac{t_1^2}{(1-\alpha)(1+\alpha)} + \frac{t_1^{\alpha+1}T^{-\alpha+1}}{(1-\alpha)(1+\alpha)}\right)\right.$$

$$\left.-\frac{d}{T^{1/n}(1+\alpha n)}\left(\frac{n}{n+1}T^{1+1/n} - \frac{T^{1+1/n}}{1-\alpha} + T^{-\alpha+1}t_1^{\alpha+1/n}\right)\right] \tag{28}$$

IX. OPTIMAL POLICIES AND PRICING OF THE MODEL:

In this paper we get the best stock process policies that are being studied. In order to find the optimal values of t_1, we compare K(t_1)'s first order partial derivatives to zero within relation to t_1. The minimum requirement for K(t_1) is

$$\frac{d^2K(t_1)}{dt_1^2} > 0$$

Differentiating K(t_1) and equating to zero with respect to t_1

$$= \frac{\lambda}{\alpha+1}\left(t_1 - \frac{4t_1}{1-\alpha} + \frac{2t_1}{(1-\alpha)(1+\alpha)} + \frac{t_1 T^{1-\alpha}}{1-\alpha}\right) - \left(\frac{d}{T^{1/n}(1+\alpha n)}\right)\left(\alpha + \frac{1}{n}\right) t_1^{\alpha+\frac{1}{n}-1} = 0 \tag{29}$$

We obtain the optimal time to stop the replenishment at t_1* of t_1 by solving the formula (29).

The optimum order quantity Q * of Q in process T is obtained by replacing the optimum value of t_1 in equation (24).

$$Q^* = \lambda t_1^* \tag{30}$$

X. NUMERICAL ILLUSTRATION OF THE MODEL:

We address numerical examples in this paper. The values of the costs and parameters associated with the model are used to illustrate the solution process of the model:

A = 2000, 2100, 2200, 2300; C = 10, 10.5, 11, 11.5; C_1 = 10, 10.5, 11, 11.5;

α = 0.5, 0.525, 0.55, 0.575, λ = 5, 5.25, 5.5, 5.75; n = 2, 2.1, 2.2, 2.3;

d = 100, 105, 110, 115; T = Twelve months.

Optimum quantity of order Q*, time of replenishment, total cost are estimated and provided in Table 3 to replace these values.

Table 3 shows that the decay and replenishment parameters have a tremendous impact on the optimum values of the model.

If cost of order ‘A’ increases from 2000 to 2300, then optimum quantity of ordering Q^* increases from 43.973 to 45.814, optimum time of replenishment t_1^* increases from 8.795 to 9.163 and the total cost K^*, decreases from 372.656 to 365.292. If cost parameter ‘C’ increases between 10 to 11.5 then optimum quantity of order Q^* increases from 43.973 to 45.052, optimum time of replenishment t_1^* increases from 8.795 to 9.01, and total cost K^*, decreases from 372.656 to 359.457. As holding cost ‘C_1’ increases between 10 to 11.5 optimum quantity of order Q^* decreases from 43.973 to 43.409, optimum time of replenishment t_1^* decreases between 8.795 to 8.682, and total cost K^*, increases from 372.656 to 409.26.

When parameter of replenishment ‘λ’ increases between 5 to 5.75 optimum quantity of order Q^* decreases from 43.973 to 40.321, optimum time of replenishment t_1^* decreases from 8.795 to 7.233, and the total cost K^*, increases from 372.656 to 402.87. The parameter of deteriorating ‘α’ increases between 0.5 to 0.575 optimum quantity of order Q^* decreases from 43.973 to 38.1, optimum time of replenishment t_1^* decreases from 8.795 to 7.458, and the total cost K^*, increases from 372.656 to 545.183.

If parameter of indexing ‘n’ increases between 2 to 2.3 then optimum quantity of order Q^* decreases between 43.973 to 36.126 optimum time of replenishment t_1^* decreases between 8.795 to 7.225, and the total cost K^*, increases from 372.656 to 448.35. As parameter of demand ‘d’ increases between 100 to 115, then optimum quantity of order Q^* decreases from 43.973 to 37.367, optimum time of replenishment t_1^* decreases between 8.795 to 7.473, and the total cost K^*, increases between 372.656 to 553.67.

Table 3
Optimum t_1 *, Q * and K * values of various parameter values

A	**C**	**C_1**	**T**	**α**	**λ**	**n**	**d**	**t_1**	**Q**	**K**
2000	**10**	**10**	12	**0.5**	**5**	**2**	**100**	8.795	43.973	372.656
2100			12					8.919	44.597	370.147
2200			12					9.042	45.211	367.693
2300			12					9.163	45.814	365.292
	10.5		12					8.868	44.338	368.185
	11.0		12					8.939	44.697	363.785
	11.5		12					9.01	45.052	359.457
		10.5	12					8.754	43.77	384.8
		11.0	12					8.716	43.582	397.004
		11.5	12					8.682	43.409	409.26
			12	**0.525**				7.776	39.288	508.819
			12	**0.550**				7.62	38.88	526.19
			12	**0.575**				7.458	38.1	545.183
			12		**5.25**			8.019	41.348	392.819
			12		**5.50**			7.235	40.152	395.8
			12		**5.75**			7.233	40.321	402.87
			12			**2.1**		8.288	41.442	399.138
			12			**2.2**		7.764	38.82	424.529
			12			**2.3**		7.225	36.126	448.35
			12				**105**	8.253	41.263	450.409
			12				**110**	7.552	37.76	543.542
			12				**115**	7.473	37.367	553.67

XI. SENSITIVITY ANALYSIS OF THE MODEL:

The analysis of sensitivity is conducted to investigate the effect on optimal policies of changes in model parameters and costs by changing each parameter (-15%, -10%,-5%, 0%, 5%, 10%, 15%) at a time for the model being studied. Table 4 summarizes the findings. Figure 4 shows the relationship between the parameters and the replenishment schedule's optimum values.

Table 4
Form Sensitivity testing – without shortages

Parameters	**Optimum policies**	**Change in parameters**						
		-15%	**-10%**	**-5%**	**0%**	**5%**	**10%**	**15%**
A	**t_1^***	8.406	8.538	8.667	8.795	8.919	9.042	9.163
	Q^*	42.03	42.69	43.337	43.973	44.597	45.211	45.814
	K^*	380.519	377.84	375.22	372.656	370.147	367.693	365.292
C	**t_1^***	8.57	8.646	8.721	8.795	8.868	8.939	9.01
	Q^*	42.848	43.228	43.603	43.973	44.338	44.697	45.052
	K^*	386.492	381.81	377.197	372.656	368.185	363.785	359.457
C_1	**t_1^***	8.839	8.887	8.839	8.795	8.754	8.716	8.682
	Q^*	44.193	44.433	44.193	43.973	43.77	43.582	43.409
	K^*	360.58	348.583	360.58	372.656	384.8	397.004	409.26
α	**t_1^***	9.051	8.945	8.925	8.795	7.776	7.62	7.458
	Q^*	46.125	45.224	44.623	43.973	37.288	38.1	38.88
	K^*	346.239	350.358	359.556	372.656	508.819	526.19	545.183
λ	**t_1^***	7.636	8.125	8.503	8.795	8.891	8.979	8.987
	Q^*	32.451	36.561	40.388	43.973	45.346	46.688	46.821
	K^*	474.066	435.126	401.54	372.656	362.283	352.53	351.588

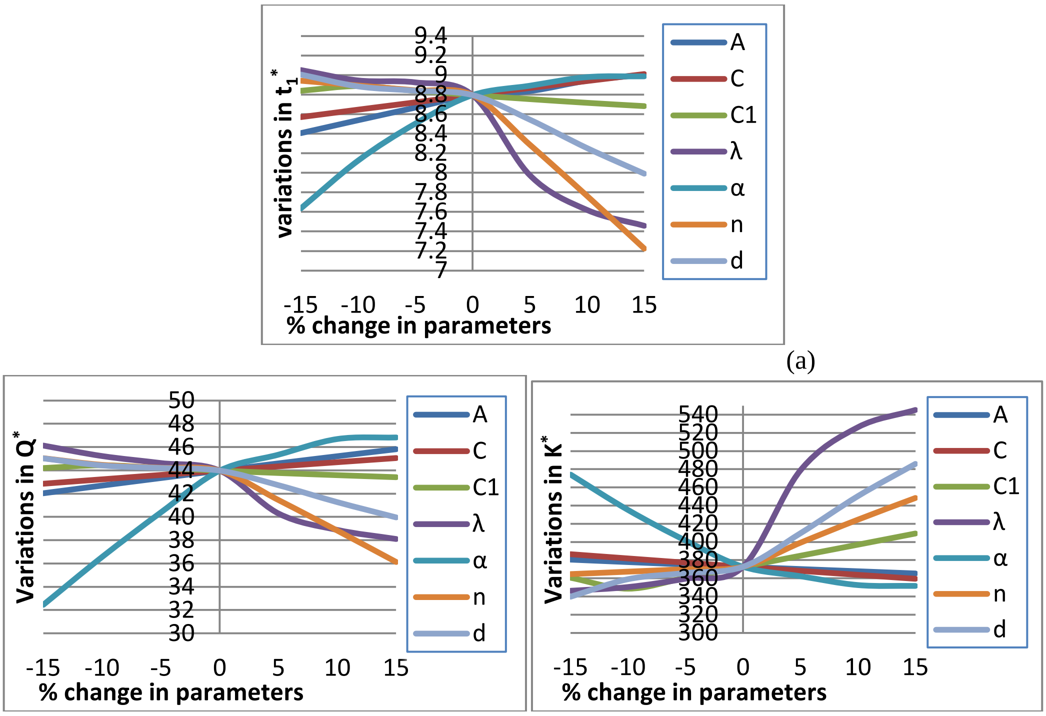
n	**t_1^***	8.942	8.893	8.844	8.795	8.288	7.764	7.225
	Q^*	44.71	44.466	44.22	43.973	41.442	38.82	36.126
	K^*	364.592	367.283	369.971	372.656	399.138	424.529	448.35
d	**t_1^***	9.005	8.884	8.84	8.795	8.544	8.253	7.991
	Q^*	45.025	44.418	44.199	43.973	42.718	41.263	39.957
	K^*	339.737	359.008	365.752	372.656	409.564	450.409	485.862

It is observed that the costs affect the optimum quantity of order and replenishment schedules significantly. As cost of order A decreases, then optimum time of replenishment t_1^* and optimum quantity of order Q^* are decreases and total cost K^* increases. As cost of order A increases, then optimum time of replenishment t_1^* and the optimum quantity of order Q^* are increases and the total cost K^* decreases. As cost per unit C decreases, optimum time of replenishment t_1^* and optimum quantity of order Q^* are decreases and total cost K^* increases. As cost per unit C increases, optimum time of replenishment t_1^* and optimum quantity of order Q^* are increases and total cost K^* decreases. When holding cost 'C_1' decreases, optimum time of replenishment t_1^* and optimum quantity of order Q^* are increases and the total cost K^* decreases. When holding cost 'h' increases, optimum time of replenishment t_1^* and optimum quantity of order Q^* are decreases and total cost K^* increases.

If replenishment parameter 'α' decreases, then optimum time of replenishment t_1^* and optimum quantity of order Q^* are increases and total cost K^* decreases. If parameter of replenishment 'α' increases, then optimum time of replenishment t_1^* and optimum quantity of order Q^* are decreases and total cost K^* increases. When parameter of deterioration 'λ' decreases, optimum time of replenishment t_1^* and optimum quantity of order Q^* are decreases and total cost K^* increases. When parameter of deterioration 'λ' increases, optimum time of replenishment t_1^* and optimum quantity of order Q^* are increases and total cost K^* decreases.

As parameter of indexing 'n' decreases, optimum values of t_1^* Q^* are increases and total cost K^* decreases. As parameter of indexing 'n' increases, optimum values of t_1^* Q^* are decreases and total cost K^* increases. If demand parameter 'd' decreases, optimum time of replenishment t_1^* and optimum quantity of order Q^* are increases and the total cost K^* decreases. If parameter of demand 'd' increases, optimum time of replenishment t_1^*, optimum quantity of order Q^* are decreases and total cost K^* increases.

It is also noted that the shortage inventory model's optimum total cost is less than the shortage-free inventory model. If the demand is a function of time, enabling back-logged shortages is rational. Historical data are generated by managers by estimating demand parameters, replenishment parameters, and deteriorating parameters to obtain optimal supply process policies. The framework also incorporates some existing models as specific cases when the replenishment distribution degenerates.



(a)

(b) (c)

Fig 4: Relationship between optimal values and parameters with shortages

XII. CONCLUSION:

A inventory model for deteriorating items with power function with time-dependent demand with Exponential deterioration level and Pareto decline had been developed in this paper. In this design, shortages were allowed and completely backlogged. A form of power feature with time-dependent demand involves a policy other than traditional Weibull-based policy. For cases where there is a large portion of the market at the beginning of the period, we use n > 1 and 0<n<1 when it is high at the end of the period. The statistical illustration and sensitivity analysis demonstrated behaviours of different parameters. It is also found that the optimum total cost of the shortage inventory model is less than that of the shortage-free stock model. For managers, the model is useful to obtain optimum supply process policies by estimating parameters of production, parameters of replenishment, and parameters of decline in historical data. The model that is proposed is more useful for analyzing the situation that occurs in areas such as dealing with Transpiration network, cement factory, food processing units sell yards or warehouses for fruit and vegetables and are also used to manage the supply chain.

Acknowledgement: This paper was developed under the UGC minor project No:MRP-7082/16 (SERO/UGC), I thank to UGC-INDIA and Director of Aditya Institute of Technology and Management, Tekkali for his continuous support of the project.

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