Summery and Conclusions

In view of the fact that with Bianchi type dark energy cosmological models have gained importance in recent years, we have discussed in chapter A general class of Bianchi Cosmological Model in the presence of a perfect fluid and non rotating dark energy is considered. A determinate solution is obtained with a special law of variation for Hubble's parameter propose by Bermann(1983) is used in a general class of Bianchi Cosmological Model in the presence of a perfect fluid and dark energy, hence non-rotaing model is obtained. An anisotropic dark energy parameter in the spatially homogeneous Bianchi-type-I with Equation of State (EoS) parameter in the frame work of Scale covariant theory is obtained . This model will help to study the role of dark energy in getting accelerated expansion of the universe popularly known as inflationary phase. Spatially homogeneous and anisotropic cosmological model play significant role in the description of large scale behavior of the universe and realistic picture of the universe in its early stages.So, a spatially homogeneous and anisotropic LRS Bianchi typeII space-time dark energy is considered in sacalar-tensor theory of gravitation proposed by Saez-Ballester(1986) obtained . The discussion of a dark energy cosmological model with Equation of State (EoS) parameter in f(R,T) gravity in Bianchi type-III space-time in the presence of a perfect fluid source. the plausible physical conditions that the scalar expansion is proposed to the shear scalar (Collins et al. 1980) and the EoS parameter is proportional to skewness parameter. It is observed that the EoS parameter and the skewness parameter in the model turn out to be functions of cosmic time which will help in discussing late - time acceleration [Reiss et al.(1998,2004)]. Spatially homogeneous and Bianchi type-V cosmological model create interest in isotropic models as special cases and allow 91 arbitrarily small anisotropy levels at any instant of cosmic time. The model obtained High dimensional dark energy cosmological model with Equation of State (EoS) parameter in scale-covariant theory of gravitation formulated by Canuto et al.(1977) in presence of perfect fluid will also help to study the structure formation of the universe in early stage and will definitely help in the discussion of accelerated expansion of the universe with special reference to scale covariant theory of gravitation. The physical and kinematical properties and their behaviors of the all above said model are discussed.

Conclusions:

1. Anisotropy dark energy models with variable EoS parameter play a prominent role in the discussion of the accelerated expansion of the universe , the crux of the problem in the present scenario.

2. The homogeneous and anisotropy General class of Bianchitype-Type dark energy model with variable EoS parameter have been investigated. It is identified that EoS parameter, skewness parameter in the model are all function of t. It can also be seen that the model is accelerating, expanding, non-rotating and free from initial singularity. Also, this model confirms that high supernova experiment.

3. The Models obtained are anisotropic and free from initial singularity.

4. Since scalar fields plays a significant role in the early stage of evolution of the universe, the models obtained and its properties throw a better light on our understanding of accelerated expansion of the universe.

5. It is observed that EoS parameter, skewness parameter in the model are all function of t. 92

6. It can also be seen that the model is accelerating, expanding, nonrotating and has no initial singularity.

7. The dark energy model obtained in this theory are model confirms that high supernova experiment.

8. we have investigate higher order Bianchi type-III dark energy model is obtained with variable EoS parameter in a scale-covariant theory of gravitation formulated by Canuto et al. [1977].

9. It is observed that the model obtained has no initial singularity and all the physical parameters are infinite at the initial epoch, t = 0 and tend to zero for large t .

10. It is also observed that the model does not approach isotropy through the whole evolution of the universe.

11. All the physical parameters of this model are diverse at the initial epoch, t = 0 and tend to zero for large values of t.

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STUDY ON BIANCHI-TYPE DARKENERGY COSMOLOGICAL MODELS IN CERTAIN MODIFIED THEORIES OF GRAVITATION

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CERTIFICATE

This is to certify that the project work titled ***" STUDY ON BIANCHI-TYPE DARKENERGY COSMOLOGICAL MODELS IN CERTAIN MODIFIED THEORIES OF GRAVITATION "*** *is carried out by me and was not submitted for partial/full financial assistance to any other funding agency.*

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Dr. Santhi Kumar Rajamahanthi
Principal Investigator

CONTENTS

CHAPTER-1

GENERAL INTRODUCTION

1.1 Introduction

We live in this universe, Hence, it is essential to understand the origin, evolution and ultimate fate of the our universe. This can be, effectively, done by constructing mathematical models of the universe, using Einstein's theory of gravitation and other modified theories of gravitation. The models, thus, obtained can be compared with the present day observations, to decide about the shape, physics, and origin of the universe. With this motivation we have taken up the investigations in this REPORT entitled **" STUDY ON BIANCHI-TYPE DARK ENERGY COSMOLOGICAL MODELS IN CERTAIN MODIFIED THEORIES OF GRAVITATION "** . This report comprises of sevent chapters and deals with some spatially homogeneous isotropic and anisotropic cosmological models of the universe in certain modified theories of gravitation.

This chapter is organized as follows: In section 1.2 we present a brief review of Einstein's theory of gravitation. Section 1.3 deals with cosmology and cosmological models of the universe. Section 1.4 is devoted to a brief introduction to Bianchi type space-times. In section 1.5, a brief discussion on dark energy is given. In section1.6, a brief review of some modified theories of gravitation is presented. The last section i.e., 1.7 deals with the problems investigated in this thesis.

1.2 EINSTEIN'S THEORY OF GRAVITATION

Einstein's theory of relativity is based on the fundamental idea of relativity of all kinds of motion. The special theory of relativity formulated by Einstein (1905) makes the restricted use of this general idea since it nearly assumes the relativity of uniform translator motion in a region of free space where gravitational effects can be neglected. As it fails to study relative motion in accelerated frames of reference and is not applicable to all kinds of motion. Taking into account of these limitations, Einstein (1915) generalized the special theory of relativity and foot forth general theory of relativity or Einstein's theory of gravitation. Einstein arrived at a novel concept which says that gravitation has a basic relationship with the space-time in which it is always present. The theory of general relativity or Einstein's relativistic theory of gravitation is a more accurate and comprehensive description of gravitation than Newtonian theory.

Einstein's theory of gravitation is based on Riemannian metric tensor g_{ij} which describes not only the gravitational field but also the geometry. This theory of gravitation has great formal beauty and mathematical elegance and is found to lead to a complete theory of gravitational action. In the development of general relativity Einstein was mainly guided by three basic principles: Principle of covariance; principle of Equivalence and Mach's principle.

Principle of covariance

This principle helps us to write the physical laws in covariant form so that their form remains unaltered in all systems of coordinates. This implies that the physical laws should be expressed in tensor form.

Principle of equivalence

This principle is the actual hypothesis by which gravitational considerations are introduced into the development of general relativity. It says that no physical experiment can distinguish whether the acceleration of a free particle is due to gravitational field or it is due to the acceleration of a frame of reference. Thus, this leads to an intimate relation between metric and gravitation. Principle of equivalence is classified into two: the strong equivalence and the weak equivalence. The strong equivalence principle states that the observable laws of nature do not depend upon the absolute values of the gravitational potentials while the weak equivalence principle implies equality of inertial and gravitational masses of a closed system. Einstein mostly used the strong equivalence principle.

Mach's Principle

The Importance of this principle is that it can be used to determine the geometry of the space – time and there by the inertial properties of a test particle from the information of the density and mass energy distribution in its neighborhood. According to this principle:

i. The inertia of a body must increase when ponder able masses are piled up in its neighborhood.
ii. A body must experience an accelerated force when neighbouring masses are accelerated and, in fact, the force must be in the same direction as that of acceleration.
iii. A rotating hollow body must generate inside of itself a "Coriolis field", which deflects moving bodies in the sense of the rotation, and a radial centrifugal field as well.

Einstein's general theory of relativity has been very successful in describing gravitational phenomena. In this theory the space – time is described by the pseudo Riemannian metric.

$$ds^2 = g_{ij}dx^i dx^j \;;\; i,j = 1,2,3 \; and \; 4 \tag{1.01}$$

and the components of the symmetric tensor g_{ij} act as gravitational potentials. The gravitational field manifests through the curvature of the space – time and the general field equation's which govern the gravitational field are given by

$$G_{ij} \equiv R_{ij} - \frac{1}{2}Rg_{ij} + \Lambda g_{ij} = -8\pi T_{ij} \tag{1.02}$$

Where G_{ij} is the Einstein tensor, R_{ij} is the Ricci Tensor, R is the Scalar curvature and T_{ij} is the energy momentum tensor due to matter and Λ is the cosmological constant (the velocity of light c in vacuum and the Newtonian Gravitational constant G are taken to be unity in this thesis). This cosmological constant was introduced by Einstein, while studying static cosmological models and was later discarded by him saying "It is the greatest blunder of my life". In this connection, it may be mentioned that, in recent years, the cosmological constant is coming into lime light and attracting many researchers in general relativity but comes as a variable and not as a constant. However we are not including Λ in our discussions.

Since the Einstein tensor G_{ij} is divergence free, the field equations (1.02) yield

$$T^{j}_{i;j} \equiv 0 \qquad (1.03)$$

which can be considered as the energy momentum conservation equation and which also gives us the equations of motion of matter.

The general theory of relativity yields results which are in good agreement to a great degree of accuracy with the experimental results. It is well known that Einstein's theory has served as a basis for the study of cosmology and cosmological models of the Universe. Hence, in the following section, we briefly describe , cosmology and cosmological models of the universe.

1.3 COSMOLOGY AND COSMOLOGICAL MODELS

Cosmology is a branch of science that deals with the study of large scale structure of the universe. The Universe consists of stars, star clusters and galaxies or the nebulae, pulsars, quasars as well as cosmic rays and background radiation. The basic problem in cosmology is the dynamics of the system. The fundamental force keeping solar systems, stars and galaxies together is the force of gravity. The other long range interactions such as electromagnetic forces may be disregarded because the galaxies, which are major constituents of the universe as well as the intergalactic medium, are known to be electrically neutral.

It is well known that Einstein's general theory of relativity is a satisfactory theory of gravitation, correctly predicting the motion of test particles and photons in curved space-time; but in order to apply to the universe one has to introduce simplifying assumptions and approximations. The first approximation that is usually made is that of continuous matter distribution. The study of cosmology is based on the cosmological principle, which states that on a sufficiently large scale the universe is homogeneous and isotropic. Physically, this implies that there is no preferred position, preferred direction or preferred epoch in the universe. Thus, by using the cosmological principle we assume that the universe is filled with a simple macroscopic perfect fluid (devoid of shear-viscous, bulk-viscous and heat conductive properties). Its energy-momentum tensor T_{ij} is, then given by

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij} \tag{1.04}$$

where ρ is its proper energy density p is the isotropic pressure and u_i is four-velocity of the fluid particles (stars etc.)

The study of the large scale structure of the physical universe is the main aim of cosmology. Cosmologists construct mathematical models of the universe and they compare these models with the present day universe as observed by astronomers. The theory of cosmological models began with Einstein's development of the static universe in 1917. In 1922, Hubble published his famous law relating to apparent luminosities of distant galaxies to their red shifts.

That is

$$V = HD \tag{1.05}$$

where V is the speed of recession of galaxy at a distance D from us and H is Hubble's constant. Because of this observed red shift of spectral lines from distant galaxies and static models of the universe were ruled out and non-static models gained importance.

Friedmann (1922) was the first to investigate the most general non-static, homogeneous and isotropic space-time described by the Robertson-Walker metric

$$ds^2 = dt^2 - R^2(t)\left\{\frac{dr^2}{1-kr^2} - r^2 d\theta^2 - r^2 \sin^2\theta d\theta^2\right\} \tag{1.06}$$

where R (t) is the scale factor, k is a constant which is by a suitable choice of r can be chosen to have values +1, 0 or -1 according as the universe is closed, flat or open respectively. He has also discussed the evolution of the function R (t) using Einstein field equations for all three curvatures.

It has been both experimentally and theoretically established that the present day universe is both spatially homogeneous and isotropic and therefore can be well described by a Friedmann-Robertson-Walker (FRW) model (Patridge and Wilkinson 1967, Ehlers et al. 1968). However, there is evidence for a small amount of anisotropy (Boughn et al 1981) and a small magnetic field over cosmic distant scales (Sofue et al. 1979). This suggests a very large departure from FRW models at early stages of evolution of the universe. Thus, it is useful to study cosmological models which may be highly anisotropic. For the sake of simplicity it is usual to restrict oneself to models that are spatially homogeneous. The spatially homogeneous and anisotropic models which are known as Bianchi models present a medium way between FRW models and completely inhomogeneous and anisotropic universes and thus play an important role in current modern cosmology. Hence, in the following section, a brief discussion of Bianchi space-times is presented.

1.4 BIANCHI MODELS

Bianchi type cosmological models are important in the sense that these are homogeneous and anisotropic in which a process of isotropization of universe is studied through the passage of time. Moreover, from the theoretical point of view anisotropic universes have a greater generality than isotropic models. Here, we present a brief description of Bianchi space-times.

Space-times admitting a three-parameter group of automorphisms are important in the discussion of cosmological models. The case where the group is simply transitive over the three-dimensional, constant-time subspace is particularly useful. Bianchi (1898) has shown that there are only nine distinct sets of structure constants for groups of this type so that the algebra may be easily used to classify homogeneous space-times. Thus, Bianchi type space-times admit a three parameter group of motions and hence have only a manageable number of degrees of freedom.

In recent years experimental studies of the isotropy of the cosmic microwave background radiation and speculation about the amount of helium formed at early stages of the universe and many other effects have stimulated theoretical interest in anisotropic cosmological models. During the last three decades there had been considerable work on anisotropic universes. The simplest of them are the well known nine types of Bianchi models (Taub, 1951) which are necessarily spatially homogeneous. Out of the nine types of Bianchi models, the only types, which can tend towards isotropy at arbitrary large times and hence, permit the formation of galaxies and the development of intelligent life, are the types-I,V.VII_0 and VII_h

(Collins and Hawking,1973). It may be recalled that the type I and VII_0 represent the generalized flat FRW models while the types V and VII_h represent the generalized open FRW models. It may also be mentioned that the types II, VI, VIII and IX represent the most general non-flat models. Between the flat and open models the study of the later type is considered which is physically more relevant because the ‘hot spots’ can occur only in these models and the positive detection of a ‘hot spot’ would unequivocally demonstrates that the universe is open (Barrow et al. 1983). In view of this, the study of Bianchi type V and VII_h models assumes considerable importance in relativistic cosmology. A complete list of all exact solutions of Einstein’s equations for the Bianchi types I – IX with perfect fluid matter is given by Krammer et al (1980).

1.5 DARK ENERGY MODELS IN GENERAL RELATIVITY

One of the outstanding developments in cosmology is the discovery of the accelerated expansion of the universe which is believed to be driven by some exotic dark energy [Perlmutter et al. (1999), Reiss et al. (1998), Spergel et al. (2003, 2007), Copeland et al. (2006)]. The nature and composition of dark energy is still an open problem. The thermo-dynamical studies of dark energy reveal that the constituent of dark energy may be mass less particles (bosons or fermions) whose collective behavior resembles with a kind of radiation fluid having negative pressure. Also, it is commonly believed by the cosmological community that this hitherto unknown exotic physical entity known as dark energy is a kind of repulsive force which acts as antigravity responsible for gearing up the universe. The Wilkinson microwave anisotropy probe (WMAP) satellite experiment suggests that dark energy is a kind of repulsive force which acts as antigravity responsible for gearing up the universe.

It has been conjectured that the simplest dark energy candidate is the cosmological constant, but it needs to be extremely fine tuned to satisfy the current value of the dark energy. Chaplygin gas as well as generalized Chaplygin gas has also been considered as possible candidates for dark energy due to negative pressure. [Srivatswa (2005); Bertolami et al. (2004); Bento et al.(2002)]. Some authors have also suggested that interacting and non-interacting two fluids Scenario are possible dark energy candidates [Setare (2007); Setate et al.(2009); Pradhan et al.(2011)]. Some have considered modified gravitational action by adding a function f(R) (R being Ricci scalar curvature)to Einstein-Hilbert Lagrangian where f(R) provides a gravitational alternative for dark energy causing late time acceleration of the universe [Nojiri & Odintsor (2003);carol et al(2004); Abdalla et al.(2005); Mena et al.(2006)]. A review on modified $f(R)$ gravity as an alternative to dark energy is made available by Nojiri & Odintsor (2007) and Copeland et al (2006). In spite of these attempts cosmic acceleration is, still, a challenge for modern cosmology.

Cosmological models based on dark energy have been widely investigated by Sami et al.(2005); Wang et al.(2007); Zimdahl & Pavon (2007); Jamil & Rashid (2008); Setare & Wagonor (2011); Li et al.(2011); these models yield stable solution of FRW equations at late times of evolving universe Farooq et at.(2011) have investigated dynamics of interacting phantom and quintessence dark energies. Pradhan et al (2011); Yadav (2011); Adhav et al.(2011) are some of the authors who have investigated Bianchi type dark energy models in general relativity.

1.6 MODIFIED THEORIES OF GRAVITATION

Since Einstein published his first theory of gravitation there has been many criticisms of general relativity. So, in recent years, several modified theories of gravitation have been proposed as alternatives to Einstein's theory. The most important among them are scalar-tensor theories of gravitation formulated by Jordan (1955), Bran's and Dicke (1961), Saez and Ballester (1986), Nordtvedt (1970), Ross (1972), Dunn (1974) and scale covariant theory of gravitation proposed by Canuto et al (1977), Schmidt et al (1981). All version of the scalar-tensor theories are based on the introduction of scalar field variable into the formulation of general relativity, this scalar field together with the metric tensor field then forms the scalar tensor field representing the gravitational field. In the scale covariant theory Einstein field equations are valid in gravitational units whereas physical quantities are measured in atomic units. The metric tensor in the two systems of units is related by a conformal transformation

$$\overline{g_{ij}} = \phi^2(x^k) g_{ij} \tag{1.10}$$

where bar denotes gravitational units an unbar denotes atomic units.

Another recent modification of Einstein theory is the generalized $f(R, T)$ gravity proposed by Harko et al (2011) to explain dark energy and accelerated expansion of the universe. In this thesis we concentrate on the investigation of some cosmological models in scalar-tensor theories of gravitation proposed by Saez and Ballester (1986), scale covariant theory of gravitation formulated by Canuto et al.(1977) and $f(R, T)$ gravity proposed by Harko et al.(2011). We now present a detailed discussion of the above modified theories of gravitation.

Bran's-Dicke theory of gravitation:

As usually formulated, Mach's principle requires that the geometry of space-time and hence the inertial properties of every infinitesimal test particle be determined by the distribution of mass-energy throughout the universe (see eg. Wheeler 1964). Although being one of the foundation stones of Einstein's philosophy, this principle is contained only to a limited extent in general relativity (Dicke 1964).Some examples of 'non-Machian' solutions are (cf. Heckmann and Schucking 1962); Minkowski space which has inertial properties but no matter; the Godel (1949) universe which contains such unphysical properties as closed time-like curves; and the closed but empty Taub (1951) model. Wheeler (1964) has suggested that these unsatisfactory solutions might be excluded by means of boundary conditions. Brans and Dicke (1961) have argued against this possibility by considering a static massive shell. The inertial properties of test particles inside shell are, according to general relativity, unchanged even if the mass of the shell is increased.

In the hope of extending general relativity in such a way as to incorporate Mach's principle, Brans and Dicke (1961) have proposed a theory which includes a long range scalar field interacting equally with all forms of matter (with the exception of electromagnetism). They noted, following Dirac (1938) and Sciama (1959), that the Newtonian gravitational constant G is related to the mass M and radius R of the visible universe by

$$G \sim \frac{Rc^2}{M} \tag{1.11}$$

(The numbers are approximate). This suggests that G is a (scalar) function determined by the matter distribution. Their theory is formally equivalent to the one previously considered by Jordan (1955).

In order to generalize the equations of general relativity, Brans and Dicke (1961) formulated their variational principle which differs from that of general relativity, namely,

$$\delta \int [R + (16G)L]\sqrt{-g}\, d^4x = 0 \tag{1.12}$$

in that G is replaced by ϕ^{-1} which now comes inside the action integral. There are also additional terms to take account of the scalar nature of ϕ and its interaction with matter. The variational principle now becomes

$$\delta \int \left[\phi R + (16\pi)L - \frac{\omega \phi_{,i}\phi^{,i}}{\phi}\right]\sqrt{-g}\, d^4 = 0 \tag{1.13}$$

where ϕ is the scalar field, $R$is the usual scalar curvature, $L$is a function of matter variables and metric tensor components (not of scalar fieldϕ) and ω is a dimensionless constant. Notation for Brans - Dicke theory is such that the metric has signature + 2, a comma denotes partial differentiation, a semi colon denotes covariant differentiation, and the velocity of light c = 1. The field equations obtained by the variation of g_{ij} and ϕ take the form

$$R_{ij} - \frac{1}{2}g_{ij}R - 8\pi\phi^{-1}T_{ij} - \omega\phi^{-2}\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,j}\phi^{,k}\right) - \phi^{-1}\left(\phi_{i;j} - g_{ij} \Box \phi\right) \tag{1.14}$$

$$(3 + 2\omega)\Box\phi = 8\pi\phi^{-1}T \tag{1.15}$$

where $\Box\phi = g^{ij}\phi_{i;j}$ and $T = g^{ij}T_{ij}$ Since the main difference between the Brans - Dicke theory and Einstein theory lies in the gravitational field equations, which determine the metric field g_{ij} rather than in the equations of motion. The energy momentum tensor of matter T_{ij} satisfies the local matter-energy conservation law.

$$T^{ij}_{;j} = 0 \tag{1.16}$$

which also represents equations of motion and are consequences of the field equations (1.14) and (1.15).

A comparison of the above equations with Einstein's equations shows that the Brans − Dicke theory goes over to general relativity in the limit

$$\omega \rightarrow \infty \ , \phi = \text{constant} = G^{-1} \tag{1.17}$$

The above modification of Einstein's theory involves violation of 'strong principle of equivalence' on which Einstein's theory is based. But this does not violate 'weak principle of equivalence', for example, the paths of test particles in a gravitational field are still independent of their masses. Thus, Brans − Dicke theory can now be described as a theory for which the gravitational force on an object is partially due to the interaction with a scalar field, and partially due to a tensor interaction.

Further discussions, by Brans and Dicke (1961), of the field equations (1.14) and (1.15) have included an analysis of the weak field equations, study of the 'three standard tests' , comparison with the work of Jordan (1955), discussions of boundary conditions for ϕ , investigations of cosmology and the general relationship to Mach's principle.

At present there is no evidence to preclude the validity of the Brans −Dicke scalar-tensor theory. While this theory does not predict an anomalous gravitational red shift, it does give values for the gravitational deflection of light rays and the perihelion advance of planetary orbits different from those of Einstein's theory (Dicke 1964, Brans − Dicke 1961).But in view of the relatively large discrepancies in the measurements of the deflection of starlight near the sun's limb during a total eclipse (Dicke 1967) and the measurements of the oblateness of the sun (Dicke and Goldenberg 1967), it is concluded that Brans− Dicke theory is not in conflict with observations, provided that $\omega \geq 6$.

Only very recently, this theory has been applied to more interesting problems in astrophysics in order to appreciate fully the implications of the addition of a long range scalar-interaction. By comparing the predictions of this theory with those of Einstein's theory, one may hope to obtain important differences which might be used to decide between the two theories. For example, while Solmona (1967) has shown that certain gross features of a cold neutron star remain unchanged by the presence of the strength of the scalar field, Morganstern and Chiu (1967) have shown that if a neutron star is observed to exhibit symmetric radial pulsations, then the existence of the scalar field may be ruled out.

As a consequence of the recent lunar ranging experiments (Williams et al 1976; Shapiro et al 1976) one can conclude that Brans-Dicke parameter $|\omega| \geq 500$. It has been pointed out that there is no theoretical reason to restrict ω to positive values (Smalley and Eby 1976).In view of this one might well conclude that Brans − Dicke theory with some large value of $|\omega|$ is the correct theory. The work of Singh and Rai (1983) gives a detailed discussion on Brans-Dicke cosmological models.

SAEZ-BALLASTER SCALAR-TENSOR THEORY

Several physically acceptable scalar-tensor theories of gravitation have been proposed and widely studied so far by many workers. Among them Brans-Dicke (1961) theory of gravitation is of considerable importance. We have seen that, in this theory, scalar field ϕ has the dimension of inverse of gravitational constant G and the role of the scalar field is confined to its effects on gravitational field equations. Saez and Ballester (1986) developed a theory in which the metric is coupled with a dimensionless scalar field. This coupling gives satisfactory descriptions of weak fields. Inspite of the dimensionless character of the scalar field an antigravity regime appears.

Saez and Ballester (1986) assumed the Lagrangian

$$L = R - w\phi^n\left(\phi_{,\alpha}\phi^{,\alpha}\right) \tag{1.18}$$

Where R is the curvature, ϕ is the dimensionless scalar field, w and n are arbitrary dimensionless constants and $\phi^{,i} = \phi_{,j}g^{ij}$. For scalar field having the dimension ϕG^{-1}, the Lagrangian given by equation (1.9) has different dimensions. However, it is a suitable Lagrangian in the case of a dimensionless scalar field.

From the Lagrangian one can build the action

$$I = \int_{\Sigma}\left(L + G\,L_m\right)\sqrt{-g}\,dx\,dy\,dz\,dt \tag{1.19}$$

where L_m is the matter Lagrangian, $g = \left|g_{ij}\right|$, Σ is an arbitrary region of integration and $G = -8\pi$. By considering arbitrary independent variations of the metric and the scalar field vanishing at the boundary of Σ, the variational principle

$$\delta I = 0 \tag{1.20}$$

leads to the Saez-Ballaster (1986) field equations for combined scalar and tensor fields given by

$$G_{ij} - w\phi^n\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) = -T_{ij} \tag{1.21}$$

$$\phi^n\phi^{,i}_{;i} + n\phi^{n-1}\phi_{,k}\phi^{,k} = 0 \tag{1.22}$$

Also ,

$$T^{ij}_{;j} = 0 \tag{1.23}$$

is a consequence of the field equations (1.20) and (1.22).

(Here we have chosen $8\pi G = c = 1$).

Saez (1987) discussed the initial singularity and inflationary universe in this theory. He has shown that there is an antigravity regime which could act either at the beginning of the inflationary epoch or before. He has also obtained non-singular FRW model in the case k=0. Also, this theory suggests a possible way to solve the missing matter problem in non-flat FRW cosmologies.

Singh and Agrawal (1991), Shri Ram and Tiwari (1998), Singh and Shri Ram (2003) and Reddy and Venkateswara Rao (2001) have investigated several aspects of cosmological models in Saez-Ballester scalar theory of gravitation with perfect fluid as source. A detailed discussion of the recent work in this theory will be presented in the relevant chapters.

SCALE COVARIANT THEORY OF GRAVITATION:

Ever since the beginning of the general relativity theory, several efforts have been devoted to construct alternative theories of gravitation. One of the most important modifications of general relativity is proposed by Brans and Dicke (1961) theory in which there exists a variable gravitational parameter G. Canuto et al. (1977a) formulated a scale – covariant theory of gravitation which also admits a variable G and which is aviable alternative to general relativity (Wesson,1980; Will,1984). In the scale-covariant theory Einstein's field equations are valid in gravitational units where as physical quantities are measured in atomic units. The metric tensors in the two systems of units are related by a conformal transformation given by the equation (1.10).where in Latin indices take values1,2,3,4 bars denote gravitational units and unbar denotes atomic quantities. The gauge functionϕ $(0 < \phi < \infty)$ in its most general formulation is a function of all space – time coordinates. Thus, using the conformal transformation of the type given by equation (1.10), Canuto et al. (1977a) transformed the usual Einstein equations into

$$R_{ij} - \frac{1}{2} R g_{ij} + f_{ij}(\phi) = -8\pi G(\phi) T_{ij} + \Lambda(\phi) g_{ij} \tag{1.24}$$

where

$$\phi^2 f_{ij} = 2\phi\phi_{i;j} - 4\phi_i\phi_j - g_{ij}\left(\phi\phi^k_{;k} - \phi^k\phi_k\right) \tag{1.25}$$

Here R_{ij} is the Ricci tensor, R the Ricci Scalar, Λ the cosmological constant, G the gravitational ‘constant’ and T_{ij}, the energy momentum tensor. A semicolon denotes covariant derivative and ϕ_i denotes ordinary derivative with respect to x^i. A particular feature of this theory is that no independent equation for ϕ exists. The possibilities that have been considered for gauge function ϕ are (Canuto et al. 1977 a, b)

$$\phi(t) = \left(\frac{t_0}{t}\right)^{\varepsilon} \quad , \quad \varepsilon = \pm 1, \pm \frac{1}{2} \tag{1.25}$$

where t_0 is constant.

The form

$$\phi \sim t^{\frac{1}{2}} \tag{1.26}$$

Is the one most favored to fit observations (Canuto et al. 1983, a, b)

Also, the energy conservation equation for perfect fluid.

$$\rho_4 + (\rho + p)u^i_{,i} = -\rho \frac{(G\phi)_4}{G\phi} - 5p\frac{\phi_4}{\phi} \tag{1.27}$$

is a consequence of the field equations (1.23) and (1.24) [equation (1.25) of Canuto et al. (1977 a,b)]. Canuto et al. (1977b), Beesham (1986a, b, c), Reddy and Venkateswarulu (1987), Reddy et al. (2002) and Reddy and Venkateswarulu (2004) have investigated several aspects of this theory of gravitation with the perfect fluid matter distribution as source. Recent work in this theory is presented in the relevant chapters.

f(R) and f(R,T) Theories of Gravity:

Recent observational data suggest that our universe is accelerating. This acceleration is explained in terms of late time acceleration of the universe and the existence of the dark matter and dark energy (Nojiri and Odintsov (2007)). The simplest dark energy candidate is the cosmological constant Λ, but it needs to be extremely fine tuned. Chapligin gas and generalized chapligin gas have also been considered as possible dark energy sources due to negative pressure(Bertolami et al (2004); Srivatsava (2005); Bento et al (2002); Bilic et al (2002); Avelino et al (2003)). Interacting and non-interacting two fluid scenario have been suggested by some authors for dark energy models (Amirhashchi et al (2011); Setare (2007); Setare (2010); Cataldo et al (2011); Katore et al (2011)). other than these approaches some authors Capozziello (2002); Caroll et al (2004); Dolgov et al (2003); Nojiri et al (2003); Nojiri et al (2004); Mena et al (2006) considered modified gravitational action by adding a function of $f(R)$ (R being the Ricci scalar curvature) to Einstein–Hilbert. Lagrangian when $f(R)$ provides a gravitational alternative for dark energy causing late time acceleration of the universe. A comprehensive review on modified $f(R)$ gravity is given by Copeland et al (2006). In spite of these attempts, late time cosmic acceleration is, still, a challenge for modern cosmology.

f(R) gravity

The basics of $f(R)$ gravity are as follow: The metric tensor plays an important role in general relativity. The dependence of Levi-Civita connection on the metric tensor is one of the main properties of general relativity. However, if we allow torsion in this theory, then the connection no longer remains the Levi-Civita connection and the dependence of connection on the metric tensor vanishes. This is the main idea behind different approaches of $f(R)$ gravity.

When the connection is Levi-Civita connection, we get metric $f(R)$ gravity. In this approach, we take variation of the action with respect to the metric tensor only. The action for $f(R)$ gravity is given by

$$S = \int \sqrt{-g}\,(f(R) + L_m)d^4x \tag{1.28}$$

where f(R) is a general function of the Ricci scalar and L_m is the matter lagrangian. The field equations resulting from this action are

$$F(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - \nabla_\mu\nabla_\nu F(R) + g_{\mu\nu}\Box F(R) = T_{\mu\nu} \tag{1.29}$$

where $F(R) = \frac{d}{dR}(f(R))$, $\Box \equiv \nabla^\mu\nabla_\nu$, and ∇_μ is the covariant derivative and $T_{\mu\nu}$ is the standard minimally coupled stress energy tensor from the lagrangian L_m. Now connecting the field equations, it follows that

$$F(R)R - 2f(R) + 3\Box F(R) = T \tag{1.30}$$

f(R,T) theory of gravity

Very recently Harko et al (2011) developed a generalized $f(R,T)$ gravity where the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar R and of the trace T of the stress energy tensor. They have obtained field equations in metric formalism. The equations motion for test particles, which follow from covariant divergence of the stress energy tensor, are also presented. They have obtained several models in this theory, corresponding to some explicit forms of the function $f(R,T)$. The gravitational field equations of this theory are obtained from the Hilbert-Einstein type variational principle.

The action for this modified theory of gravity is given by

$$S = \frac{1}{16\pi G}\int (f(R,T) + L_m)\sqrt{-g}d^4x \tag{1.31}$$

where $f(R,T)$ is an arbitrary function of the Ricci scalar R and of the trace T of the stress energy tensor of matter and L_m is the matter Lagrangian. The stress energy tensor of matter is

$$T_{\mu\upsilon} = \frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_m)}{\delta g^{\mu\upsilon}} \tag{1.32}$$

Using gravitational units $(G = c = 1)$ the corresponding field equations of $f(R,T)$ gravity are obtained, by varying the action (1) with respect to the metric $g_{\mu\upsilon}$ as

$$f_R(R,T)R_{\mu\upsilon} - \frac{1}{2}f(R,T)g_{\mu\upsilon} + \left(g_{\mu\upsilon}\Box - \nabla_\mu\nabla_\upsilon\right)f_R(R,T) = 8\pi T_{\mu\upsilon} - f_T(R,T)\Theta_{\mu\upsilon} \tag{1.33}$$

where

$$f_R = \frac{\delta f(R,T)}{\delta R}, f_T = \frac{\delta f(R,T)}{\delta T}, \Theta_{\mu\upsilon} = g^{\alpha\beta}\frac{\delta T_{\alpha\beta}}{\delta g^{\mu\upsilon}}, \Box = \nabla^\mu\nabla_\mu \tag{1.34}$$

Here ∇_μ is the covariant derivative and $T_{\mu\nu}$ is usual matter energy-momentum tensor derived from the LagrangianL_m. It can be observed that when $f(R,T) = f(R)$, then equations (1.33) reduce to field equations of $f(R,T)$ gravity proposed by Nojiri and Odintsov (2003).

Contraction of equation (1.33) yields

$$f(R,T)R + 3\Box f_R(R,T) - 2f(R,T) = 8\pi T - f_T(R,T)T - f_T(R,T)\Theta \tag{1.35}$$

Harko et al (2011) obtained some particular classes of $f(R,T)$ modified gravity models by specifying functional forms of $f(R,T)$ as

$$\left.\begin{array}{ll} (i) & f(R,T) = R + 2f(T) \\ (ii) & f(R,T) = f_1(R) + f_2(T) \\ (iii) & f(R,T) = f_1(R) + f_2(R)f_3(T) \end{array}\right\} \quad (1.36)$$

Generally, the field equations also depend through the tensor $\Theta_{\mu\nu}$, on the physical nature of the matter field. Depending on the nature of the matter source for each choice of f, we can obtain several theoretical models of $\mathrm{f(R,T)}$ gravity. Harko et al (2011) have discussed FRW cosmological models in this theory by choosing appropriate function $\mathrm{f(T)}$. They have also discussed the case of scalar fields, since scalar fields play a significant role in cosmology. The equations of motion of test particles and a Brans-Dicke (1961) type formulation of the model are also presented. More details about the field equations of this theory and the research work carried out in this theory is presented in the relevant chapters where in the problems pertaining to this are investigated.

1.7 PROBLEMS INVESTIGATED

This section is devoted to the discussion of the problems investigated and the results obtained in this report.

In view of the fact that with Bianchi type dark energy cosmological models have gained importance in recent years, we have discussed in chapter 2, a general class of Bianchi Cosmological Model in the presence of a perfect fluid and non rotating dark energy is considered. To obtain a determinate solution a special law of variation for Hubble's parameter propose by Bermann(1983) is used. Some physical and Kinematical properties of the model are also discussed. The model obtained here will definitely help in the discussion of accelerated expansion of the universe

In chapter 3, we have investigated the evolution of anisotropic dark energy parameter in the spatially homogeneous Bianchi-type-I with Equation of State (EoS) parameter in the frame work of Scale covariant theory . The physical and kinematical properties of the model presented will help to study the role of dark energy in getting accelerated expansion of the universe popularly known as inflationary phase.

In chapter 4, a spatially homogeneous and anisotropic LRS Bianchi type-II space-time dark energy is considered in sacalar-tensor theory of gravitation proposed by Saez-Ballester(1986). Spatially homogeneous and anisotropic cosmological model play significant role in the description of large scale behavior of the universe and realistic picture of the universe in its early stages. The physical and kinematical properties of the model are discussed. The model obtained will also help to study the structure formation of the universe in early stage.

In chapter 5, is devoted to the discussion of a dark energy cosmological model with Equation of State (EoS) parameter in f(R,T) gravity in Bianchi type-III space-time in the presence of a perfect fluid source. To obtain a determinate solution a special law by variation for Hubble's parameter proposed by Bermann(1983) is used. We have also used the plausible physical conditions that the scalar expansion is proposed to the shear scalar (Collins et al. 1980) and the EoS parameter is proportional to skewness parameter. It is observed that the EoS parameter and the skewness parameter in the model turn out to be functions of cosmic time which will help in discussing late - time acceleration [Reiss et al.(1998,2004)]. Some physical and kinematical parameters of the model also discussed.

In chapter 6, a spatially homogeneous and dark energy Bianchi type-V dark energy is considered in sacalar-tensor theory of gravitation proposed by Saez-Ballester (1986). The physical and kinematical properties of the model are discussed . The model obtained Spatially homogeneous and Bianchi type-V cosmological model create interest in isotropic models as special cases and allow arbitrarily small anisotropy levels at any instant of cosmic time.

In chapter 7, is devoted to the discussion of a Higher dimensional dark energy cosmological model with Equation of State (EoS) parameter in scale-covariant theory of gravitation formulated by Canuto et al.(1977) in presence of perfect fluid. The physical and kinematical properties of the model are discussed. The model obtained will also help to study the structure formation of the universe in early stage and will definitely help in the discussion of accelerated expansion of the universe with special reference to scale covariant theory of gravitation.

The origin and structure of the universe is one of the greatest cosmological mysteries even today. So the aim of cosmology is to determine the large scale structure of the physical universe. This can be effectively done by constructing cosmological models of the universe using Einstein's theory of gravitation. Several cosmological models with various physical situations have been constructed by many authors and exist in literature. Since the discovery of general relativity by Einstein there have been numerous modifications of it. With the advent of modified theories of gravitation, construction of cosmological models have gained importance in this theories for a better understanding of the universe, because we are living in this universe. In particular spatially homogeneous anisotropic and isotropic cosmological models presented, in this report, in certain modified theories of gravitation will play a vital role in understanding a structure formation and the accelerated expansion of the universe with special reference to modified theories of gravitation.

1.8 Notations used in this technical report

S.No.	Physical quantity	Notation
1	Spatial Volume	V
2	Scalar of expansion	θ
3	Shear scalar	σ^2
4	Hubble's parameter	H
5	Energy density	ρ
6	String tension density	λ
7	Coefficient of bulk viscosity	ζ
8	Energy density of the particles attached to the string	ρ_p
9	The deceleration parameter	q
10	Anisotropy parameter	A_h
11	Proper pressure	p
12	Bulk viscous pressure	$\bar{p}$
13	Cosmological constant	Λ
14	Scalar field	$\emptyset$

CHAPTER-2

General Bianchi Type Dark Energy Cosmological model in Modified theories of Gravitation

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2.1 INTRODUCTION

Alternative theories of gravity draw high attention of research works, in recent past years, by studying the strong evidence of late time accelerated expansion of the universe which comes from high red shift supernova experiment(Riess et al.[1998,2004], Perlmutter et al.[1999], Bennett [2003] . The general class of Bianchi-Type models to provide natural gravitational alternatives to dark energy.

Dark energy models in general relativity and in alternative theories of gravity have recently become a prominent subject to make the research further for several authors (Sahni and Starobinsky [2000], Padmanabhan [2003], Caldwell [2002], Nojiri and Odintsov [2003], Kamenshchik et al. [2001] wang et al.[2005], Setare [2006], Naidu et al. [2012], Rao et al. [2012]). In Dark energy , the equation of state (EoS) parameter is defined by $\omega(t) = \frac{p}{\rho}$ is a function of cosmic time t , where p and ρ are pressure and density of the model respectively. The understanding of the EoS parameter, □ being function of cosmic time t is to distinguish several dark energy models and to characterize the dark energy models conventionally . Caroll and Hoffman [2003], Ray et al. [2010], Akarsu and Kilinc [2010],Yadav and yadav [2011], Pradhan et al.[2011], Amirhashchi [2011] have made their on dark energy models with variable EoS parameter.

Spatially homogeneous and anisotropic cosmological models have become very prominent in the research study of large scale structure of the universe, and such models have been studied in general relativity to understand the picture of the universe in it's early stages. Yadav et al. [2007], Pradhan et al. [2011] have researched scholarly homogeneous and anisotropic Bianchi type-III space time in the context of massive strings. Yadav and Yadav [2010] have obtained Bianchi type-II anisotropic dark energy model with constant deceleration parameter. Pradhan and Amirhashchi [2011] have investigated a new anisotropic Bianchi type-III dark energy model, in general relativity, with equation of state (EoS) parameter without assuming constant deceleration parameter. Reddy et al. [2012, 2013, 2014] has presented Bianchi type-II and III dark energy models in Saez-Bellaster scalar-tensor , f(R,T) theory of gravitation. Shriram et al.(2013) has presented general class of Bianchi type model with f(R,T) theory of gravitation.

Inspired by the above works, we have investigated in this paper General class of Bianchi-Type anisotropic dark energy cosmological model with variable EoS parameter .

This paper is organized consisting the following sections. In the Section-2.2, the metric and the field equations. In the Section-2.3 Solution of the field equations. In the Section-2.4 Some physical properties, and in the Section -2.5 contains conclusions.

2.2. METRIC AND FIELD EQUATIONS

The diagonal form of the general class of Bianchi Cosmological is

$$ds^2 = dt^2 - A^2(t)dx^2 - e^{-2x}B^2(t)dy^2 - C^2e^{-2mx}(t)dz^2 \qquad (2.01)$$

where A, B, and C are cosmic scale factors and m is a positive constant.

If m=0 , the metric (2.01) represents Bianchi type-III model

If m= -1 , the metric (2.01) represents Bianchi type-VI_0 model

If m= 1 , the metric (2.01) represents Bianchi type-V model

If m=h-1 , the metric (2.01) represents Bianchi type-VI_h model

In natural units ($8\pi G = 1, c = 1$), the Einstein field equations in case of dark energy components take the form

$$G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij} \qquad (2.02)$$

The energy momentum tensor for dark energy and perfect fluid is

$$T_j^i = (\rho_{tot} + p_{tot})u_iu_j - p_{tot}g_{ji} = (\rho + p)u_iu_j - pg_{ji} \qquad (2.03)$$

where u^i co-moving vectors such that $u^i = (0,0,0,1) and\ u_i u^j = 1$

The non-vanishing components of energy momentum tensor are ,

$$T_1^1 = T_2^2 = T_3^3 = -p_{tot}\ and\ T_4^4 = \rho_{tot} \quad (2.04)$$

Here $\rho = \rho_{tot} = \rho_{DE} + \rho_{PF}$, $p = p_{tot} = p_{DE} + p_{PF}$

Where ρ is energy density , p is pressure and of the fluid

The equation of EoS parameter of the fluid is $\omega = \frac{p}{\rho}$ (2.05)

Now assuming co-moving coordinate system, the field equations for the metric (2.01) by Eqs. (2.02) , (2.04), and (2.05) , we obtain

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\frac{\dot{A}}{A} + \frac{\dot{C}}{C}\frac{\dot{B}}{B} - \frac{m^2+m+1}{A^2} = -\rho \quad (2.06)$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{C}}{C}\frac{\dot{B}}{B} - \frac{m}{A^2} = p \quad (2.07)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{C}}{C}\frac{\dot{A}}{A} - \frac{m^2}{A^2} = p \quad (2.08)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{B}}{B}\frac{\dot{A}}{A} - \frac{1}{A^2} = p \quad (2.09)$$

$$(m+1)\frac{\dot{A}}{A} - \frac{\dot{B}}{B} - m\frac{\dot{C}}{C} = 0 \quad (2.10)$$

Where an overhead dot denotes differentiation with respect to t.

The spatial volume is

$$V = ABC = R^3 \quad (2.11)$$

Here, R(t) is the scale factor

The Hubble's parameter H is

$$H = \frac{\dot{R}}{R} = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) \quad (2.12)$$

where $H_x = \frac{\dot{A}}{A}$, $H_y = \frac{\dot{B}}{B}$ $H_z = \frac{\dot{C}}{C}$

The scalar expression θ and the shear scalar σ , mean anisotropy parameter for the metric (2.01), are defined as

$$\theta = 3\left(\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)\right) \tag{2.13}$$

$$\sigma^2 = \frac{1}{2}\left((\frac{\dot{A}}{A})^2 + (\frac{\dot{B}}{B})^2 + (\frac{\dot{C}}{C})^2\right) - \frac{1}{6}\theta^2 \tag{2.14}$$

$$A_\alpha = \frac{1}{3}\sum_{i=1}^{3}(\frac{\Delta H_i}{H})^2 \tag{2.15}$$

where $\Delta H_i = H_i - H$

The constant deceleration parameter q is defined by

$$q = -\frac{R\ddot{R}}{\dot{R}^2} = -(kt+n-1) \tag{2.16}$$

2.3. SOLUTION OF THE FIELD EQUATIONS

The inter dependence relation of q and H is

$$q+1 = \frac{d}{dt}(\frac{1}{H}) \tag{2.17}$$

By the Equation .(2.16), The scale factor R(t) obtained by

$$R(t) = e^{\delta}(e^{\int\frac{dt}{\int(1+t)dt+r}}) \tag{2.18}$$

Where δ *and* r are arbitrary constants ,

By Abdussattar, e.t.al,(2011), the proposed choice of q is

$$q+1 = \frac{-\alpha}{t^2}+\beta \tag{2.19}$$

where α is a parameter and β is constant,

Depending on different values of α and β , the defined models may vary

Without loss of generality, we consider $\delta = 0$ *and* $r = 0$

We obtain the scale factor R(t) as

$$R(t) = \left(t^2+\frac{\alpha}{\beta}\right)^{1/2\beta} \tag{2.20}$$

Integrating Eqs.(2.10) , The relation among A , B and C is

$$A^{m+1} = BC^{m} \tag{2.21}$$

It is difficult to calculate the exact values of the unknowns A, B , C , p and ρ from (2.06) to (2.09) .

Since ,the average scale factor and the spatial volume V are proportional to each other and the relation between them is

$$V = R(t)^3 = ABC \tag{2.22}$$

Clearly V is directly propositional to A, B and C respectively. To find the values of A, B and C , to assume $C = V^K$, where K is constant, in (2.10) and (2.11)

The metric coefficients for the field equations are

$$A = \left(t^2 + \frac{\alpha}{\beta}\right)^{k_1} \tag{2.23}$$

$$B = \left(t^2 + \frac{\alpha}{\beta}\right)^{k_2} \tag{2.24}$$

$$C = \left(t^2 + \frac{\alpha}{\beta}\right)^{k_3} \tag{2.25}$$

Where $k_1 = \frac{3m+3mK-3K}{2\beta(m+2)}$, $k_1 = \frac{3m-3mK-3K-3m^2K}{2\beta(m+2)}$, $k_1 = \frac{3K}{2\beta}$

With the help of (2.23),(2.24) and (2.25) the metric (2.01) will become

$$ds^2 = dt^2 - \left(t^2 + \frac{\alpha}{\beta}\right)^{2k_1} dx^2 - e^{-2x}\left(t^2 + \frac{\alpha}{\beta}\right)^{2k_2} dy^2 - \left(t^2 + \frac{\alpha}{\beta}\right)^{2k_3} e^{-2mx}(t)dz^2 \tag{2.26}$$

This model is represents similar form of general class of Bianchi type dark energy model

2.4. S0ME PHYSICAL PROPERTIES OF THE MODEL

Equation (2.26) represents the general class of Bianchi type dark energy model. The following physical and kinematical parameters of the model

$$V = R(t)^3 = ABC = \left(t^2 + \frac{\alpha}{\beta}\right)^{k_4} \tag{2.27}$$

Where $k_4 = k_1 + k_2 + k_3$

$$H = \frac{1}{3}(H_1 + H_2 + H_3) = k_5\left(\frac{t}{t^2+\frac{\alpha}{\beta}}\right) \tag{2.28}$$

where $H_1 = \frac{\dot{A}}{A} = 2k_1\left(\frac{t}{t^2+\frac{\alpha}{\beta}}\right) H_2 = 2k_2\left(\frac{t}{t^2+\frac{\alpha}{\beta}}\right),\ H_3 = \frac{\dot{C}}{C} = 2k_3\left(\frac{t}{t^2+\frac{\alpha}{\beta}}\right)$

and $k_5 = \frac{2}{3}(k_1 + k_2 + k_3) = \left(\frac{2+Km-Km^2}{\beta(m+2)}\right)$.

$$\theta = 3\left(\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)\right) = 3H = k_6\left(\frac{t}{t^2+\frac{\alpha}{\beta}}\right) \tag{2.29}$$

Where $k_6 = 3k_5 = \left(\frac{3+3Km-3Km^2}{\beta(m+2)}\right)$

$$\sigma^2 = \frac{1}{2}\left((\frac{\dot{A}}{A})^2 + (\frac{\dot{B}}{B})^2 + (\frac{\dot{C}}{C})^2\right) - \frac{1}{6}\theta^2 = k_7\left(\frac{t^2}{\left(t^2+\frac{\alpha}{\beta}\right)^2}\right) \tag{2.30}$$

Where $k_7 = 2\left({k_1}^2 + {k_2}^2 + {k_3}^2\right) - \frac{{k_6}^2}{6}$

$$A_\alpha = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 = \left[\frac{4\left({k_1}^2+{k_2}^2+{k_3}^2\right)}{3{k_5}^2} - 1\right] = \text{Constant} \tag{2.31}$$

$$q = -\frac{R\ddot{R}}{\dot{R}^2} = -(kt + n - 1) \tag{2.32}$$

The energy density in the model is

$$\rho = -\left[(k_1.k_2 + k_2.k_3 + k_3.k_1)\left(\frac{4t^2}{\left(t^2+\frac{\alpha}{\beta}\right)^2}\right) - \frac{(m^2+m+1)}{\left(t^2+\frac{\alpha}{\beta}\right)^{2k_1}}\right] \tag{2.33}$$

The pressure in the model is

$$p = \left[(a_2 + a_3)\left(\frac{t^2}{\left(t^2+\frac{\alpha}{\beta}\right)^2}\right) + k_2.k_3\left(\frac{4t^2}{\left(t^2+\frac{\alpha}{\beta}\right)^2}\right) - \frac{(m)^2}{\left(t^2+\frac{\alpha}{\beta}\right)^{2k_1}} \right] \tag{2.34}$$

Where $a_2 = k_2(k_2 - 1)$ and $a_3 = k_3(k_3 - 1)$ and

$$k_1 = \frac{3m+3mK-3K}{2\beta(m+2)}, k_1 = \frac{3m-3mK-3K-3\ \ {}^2K}{2\beta(m+2)}, k_1 = \frac{3K}{2\beta}$$

The Equation of state (EoS) parameter given by

$$\omega(t) = \frac{p}{\rho} = \frac{-\left[(a_2+a_3)\left(\frac{t^2}{\left(t^2+\frac{\alpha}{\beta}\right)^2}\right)+k_2.k_3\left(\frac{4t^2}{\left(t^2+\frac{\alpha}{\beta}\right)^2}\right)-\frac{(m)^2}{\left(t^2+\frac{\alpha}{\beta}\right)^{2k_1}}\right]}{\left[(k_1.k_2+k_2.k_3+k_3.k_1)\left(\frac{4t^2}{\left(t^2+\frac{\alpha}{\beta}\right)^2}\right)-\frac{(m^2+m+1)}{\left(t^2+\frac{\alpha}{\beta}\right)^{2k_1}}\right]} \tag{2.35}$$

The skewness parameter is given by

$$\delta = -\omega(t) = \frac{\left[(a_2+a_3)\left(\frac{t^2}{\left(t^2+\frac{\alpha}{\beta}\right)^2}\right)+k_2.k_3\left(\frac{4t^2}{\left(t^2+\frac{\alpha}{\beta}\right)^2}\right)-\frac{(m)^2}{\left(t^2+\frac{\alpha}{\beta}\right)^{2k_1}}\right]}{\left[(k_1.k_2+k_2.k_3+k_3.k_1)\left(\frac{4t^2}{\left(t^2+\frac{\alpha}{\beta}\right)^2}\right)-\frac{(m^2+m+1)}{\left(t^2+\frac{\alpha}{\beta}\right)^{2k_1}}\right]} \tag{2.36}$$

It may be observed that the general class of Bianchi Type with dark energy cosmological model is free from initial singularity, i.e. at t=0. The spatial volume in this model enhances to large time causes expansion of universe i.e., the accelerated accelerated expansion of the universe occurred . It can also be identified that $H, \theta, \sigma, \delta, \omega\ and\ p$ are functions of t and vanish for large t which they diverge at early stage of the universe.

2.5 CONCLUSIONS

We came to observe here that anisotropy dark energy models with variable EoS parameter play a prominent role in the discussion of the accelerated expansion of the universe , the crux of the problem in the present scenario. Here homogeneous and anisotropy General class of Bianchitype-Type dark energy model with variable EoS parameter have been investigated. It is identified that EoS parameter, skewness parameter in the model are all function of t. It can also be seen that the model is accelerating, expanding, non-rotating and free from initial singularity. Also, this model confirms that high supernova experiment. The model obtained in this theory is similar to the Bianchi type anisotropic dark energy model and its behavior obtained by Shriram et.al., (2013).

CHAPTER-3

Anisotropic Dark Energy Model in Alternative Theory of Gravitation

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3.1.Introduction

The alternative theories of gravitation are currently playing a prominent role in researching and analyzing the creation of systems and universe evaluation. In general relativity, the scale-covariant theory of gravitation formulated by researchers is one of the alternative theories (Reddy rt al. 2012,2014).We've seen the variable gravitational constant G, which is also part of the scale-covariant theory. Within Einstein's field equations, gravitational units are valid and atomic units are valid for physical quantity. With conformal transformation, the metric tensors in the two unit systems are associated

$$\overline{g_{ij}} = \phi^2(x^k) g_{ij} \tag{3.01}$$

Here the Latin indices are 1,2,3,4. The bar and unbar indicates gravitational units, and atomic units respectively .
Accordingly, by the conformal mapping of the type in equation (3.01), the transformed equations of Einstein are by Canuto et al (1977).

$$R_{ij} - \frac{1}{2} R\, g_{ij} + f_{ij}(\phi) = -8\,\pi\, G(\phi)\, T_{ij} + \Lambda\,(\phi)\, g_{ij} \tag{3.02}$$

Here,

$$\phi^2 f_{ij} = 2\,\phi\,\phi_{i;j} - 4\,\phi_i\,\phi_j - g_{ij}\left(\phi\phi^{,k}_{;k} - \phi^k\phi_k\right) \tag{3.03}$$

Here, T_{ij} is the energy momentum tensor, R_{ij} is the Ricci tensor, Λ is cosmological constant, , R is the Ricci scalar and G is the gravitational 'constant'.

The semi colon indicates covariant derivative, ϕ_i denotes ordinary derivative with respect to x^i. The gauge function ϕ $(0 < \phi < \infty)$ is dependent and a function of all space-time coordinates also defined as

$$\phi(t) = \left(\frac{k_0}{t}\right)^{\epsilon}, \quad \varepsilon = \pm 1, \quad \pm\frac{1}{2} \tag{3.04}$$

Here, k_0 is constant.

$$\phi \sim t^{\frac{1}{2}} \quad \text{implies that} \quad \phi = \phi_0\, t^{\frac{1}{2}} \tag{3.05}$$

is the best fit to observations.

It is common knowledge that, the mysterious dark energy has driven the rapid expansion of the universe. It influences the characterization of dark energy model with the EoS parameter $\omega(t) = \frac{p}{\rho}$ is not necessarily constant, where p is the pressure and ρ is the energy density of the fluid . Significant introduction as well as excellent review of dark energy models in general relativity is given by several authors (Reddy et al .(2013a,2013b)) in recent years. A dark energy model with axially symmetrical Bianchi type-I has been probed in the scale-covariant theory of gravitation.

This chapter consisting the following sections : (3.2) metric and field equations (3.3) Solution of the field equations. And (3..4)the physical and kinematical behavior of the model . (3.5) conclusions.

3.2.Metric and Field Equations

Consider an axially symmetric spatially homogeneous Bianchi type-I metric equation is

$$ds^2 = dt^2 - {A_1}^2\, dx^2 - {B_1}^2\,(dy^2 + dz^2) \tag{3.06}$$

Here, A_1 and B_1 are functions of cosmic time t

The energy movement tensor is

$$T_j^i = \text{diag}\ [T_0^0, T_1^1, T_2^2, T_3^3\ ] \tag{3.07}$$

We can parameterize as follows

$$\left.\begin{aligned} T^i_j &= \mathrm{diag}[\rho, -p_x, -p_y, -p_z\ ] \\ &= \mathrm{diag}[1, -\omega_x, -\omega_y, -\omega_z\ ]\rho \\ &= \mathrm{diag}[1, -\omega, -(\omega+\delta), -(\omega+\gamma)]\rho \end{aligned}\right\} \tag{3.08}$$

In equation (3.08) ρ represents the energy density of the fluid with pressures and EoS parameters along cosmic coordinates are p_x, p_y, p_z and $\omega_x, \omega_y, \omega_z$ respectively .

The equation of state EoS parameter is $\omega = \frac{\rho}{p}$ (3.09)

Since we parameterized the deviation from isotropy by choosing $\omega_x = \omega$ and then skewness parameters δ and γ are introduced. Since the equation (3.06) is axially symmetric spatially homogeneous, $T^2_2 = T^3_3$. Hence, we consider $\delta = \gamma$.

In a co-moving coordinate system the field equations (3.02) and (3.03) using the equations (3.07) , (3.08) and (3.09) , we have

$$2\frac{\dot{A_1}\dot{B_1}}{A_1B_1} + \left(\frac{\dot{B_1}}{B_1}\right)^2 - \frac{\ddot{\phi}}{\phi} + \frac{\dot{A_1}\dot{\phi}}{A_1\phi} + 2\frac{\dot{B_1}\dot{\phi}}{B_1\phi} + 3\left(\frac{\dot{\phi}}{\phi}\right)^2 = 8\pi G\rho \tag{3.10}$$

$$2\frac{\ddot{B_1}}{B_1} + \left(\frac{\dot{B_1}}{B_1}\right)^2 + \frac{\ddot{\phi}}{\phi} - \frac{\dot{A_1}\dot{\phi}}{A_1\phi} + 2\frac{\dot{B_1}\dot{\phi}}{B_1\phi} - \left(\frac{\dot{\phi}}{\phi}\right)^2 = -8\pi G\omega\rho \tag{3.11}$$

$$\frac{\ddot{B_1}}{B_1} + \frac{\ddot{A_1}}{A_1} + \frac{\dot{A_1}\dot{B_1}}{A_1B_1} + \frac{\ddot{\phi}}{\phi} + \frac{\dot{A_1}\dot{\phi}}{A_1\phi} - \left(\frac{\dot{\phi}}{\phi}\right)^2 = -8\pi G\rho(\omega+\gamma) \tag{3.12}$$

Here the overhead dot indicates derivative with respect to t.

The spatial volume

$$V = A_1 {B_1}^2 \tag{3.13}$$

The average scale factor

$$R(t) = \left(A_1 {B_1}^2\right)^{\frac{1}{3}} \tag{3.14}$$

The conventional deceleration parameter q

$$q = -\frac{R\ddot{R}}{(\dot{R})^2} \tag{3.15}$$

The scale expansion

$$\theta = \frac{\dot{A_1}}{A_1} + 2\frac{\dot{B_1}}{B_1} \tag{3.16}$$

The shear scalar

$$\sigma^2 = \frac{1}{3}\left(\frac{\dot{A_1}}{A_1} - \frac{\dot{B_1}}{B_1}\right)^2 \tag{3.17}$$

The average anisotropy parameter

$$A_l = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 \tag{3.18}$$

where $\Delta H_i = H_i - H \quad (i = 1,2,3)$

The Hubble's parameter

$$H = \frac{\dot{R}}{R} = \frac{1}{3}\left(\frac{\dot{A_1}}{A_1} + 2\frac{\dot{B_1}}{B_1}\right) \tag{3.19}$$

$$H = \frac{1}{3}(H_1 + H_2 + H_3) \tag{3.20}$$

Here, $H_1 = \frac{\dot{A_1}}{A_1}$, $H_2 = H_3 = \frac{\dot{B_1}}{B_1}$, are the directional Hubble's parameters in the directions of x, y, z respectively.

3.3. Solutions of the field equations

The equations (10)-(12) are a highly non-linear system of differential equations with 5 unknown equations. In order to obtain explicit solutions to the system, two additional constraints relating to these parameters are required. Clearly , by R Chaubey et al (2017)), the decelerating parameter q is linear in time with a negative slop, and the decelerating parameter q non linear with positive slop and it is a function of time t.

The linear varying deceleration parameter is

$$q = -\frac{R\ddot{R}}{(\dot{a})^2} = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = -\left(\frac{\dot{H}}{H} + 1\right) = -kt + n - 1 \tag{3.21}$$

In equation (3.21) k and n are (>) constants.

If q is positive i.e., $n > 1 + kt$, $q > 0$ then the model stands for decelerating universe.

If q is negative i.e., $kt < n < 1 + kt$, $-1 < q < 0$ then the model stands for accelerating universe .

From (3.21) , the average scale factor 'R' is

$$R = (n\,l\,t + c_1)^{\frac{1}{n}} , \quad k = 0, \; n > 0 \tag{3.22}$$

$$R = c_2\, e^{lt} \quad k = 0 \;\; n = 0 \tag{3.23}$$

$$R = c_3\, e^{\frac{2}{n}\tanh^{-1}\left(\frac{kt}{n} - 1\right)} , \quad k > 0, \;\; n > 1 \tag{3.24}$$

where $n = (q + 1 + kt) > 0$, c_1 , c_2 , c_3 are constants

Case1. *If* $k = 0$, $n > 0$ and $A_1 = V^M$, here M is constant

$$A_1 = (nlt + c_1)^{\frac{3M}{n}}, \qquad B_1 = (nlt + c_1)^{\frac{3M(1-M)}{2n}} \tag{3.25}$$

Hence the metric (3.06) is

$$ds^2 = dt^2 - (nt)^{\frac{3M}{n}} dx^2 - (nt)^{\frac{3M(1-M)}{2n}} (dy^2 + dz^2) \tag{3.26}$$

with out loss of generality take $l=1$ and $c_1 = 0$

3.4 Physical And Kinematical Properties of The Model

When $k = 0,\ n > 0$

The metric equation (3.26) represents an axially symmetric Bianchi type-I dark energy cosmological model

The energy density

$$\rho = \frac{1}{8\ \pi\ G}\left[\left(\frac{9\ M\ (1-M)}{n^2}\right) + \left(\frac{3\ M\ (1-M)}{2n}\right)^2 - \left({\phi_0}^2\epsilon^2 - \phi_0\epsilon\right) + \left(\frac{3\phi_0\epsilon}{n}\right) + \left(\frac{3\phi_0\epsilon M\ (1-M)}{n}\right) + 3\left({\phi_0}^2\epsilon^2\right)\right]\left(\frac{1}{t^2}\right) \tag{3.27}$$

The EoS parameter is

$$\omega = -\frac{1}{8\pi G\rho}\left[3M(1-M)\left(\frac{3M(1-M)-1}{2n^2}\right) + \left({\phi_0}^2\epsilon^2 - \phi_0\epsilon\right) - \left(\frac{3\phi_0\epsilon}{n}\right) + \left(\frac{3\phi_0\epsilon M\ (1-M)}{n}\right) - \left({\phi_0}^2\epsilon^2\right)\right]\left(\frac{1}{t^2}\right) \tag{3.28}$$

The Skewness parameter is

$$\gamma = \delta = \frac{-1}{8\pi G\rho}\left[6M(1-M)\left(\frac{3M(1-M)-n}{2n^2}\right) + 3M\left(\frac{3M-n}{n^2}\right) + \left(\frac{3M(1-M)}{2n}\right)^2 + 2\left(\frac{3\phi_0\epsilon M\ (1-M)}{n}\right)\right]\left(\frac{1}{t^2}\right) \tag{3.29}$$

$$\phi = \phi_0 t^{\epsilon}, \qquad \epsilon = \pm 1, \pm\frac{1}{2} \tag{3.30}$$

$$V = (n\,t)^{\frac{3}{n}} \tag{3..31}$$

$$\theta = \left(\frac{3(2M - M^2)}{n}\right)\frac{1}{t} \tag{3.32}$$

$$\sigma^2 = \left(\frac{9(M + M^2)^2}{n^2}\right)\left(\frac{1}{t^2}\right) \tag{3.33}$$

$$A_l = \frac{-6M^4 + 18M^3 - 15M^2}{2M^2(2 - M)^2} \qquad (3.34$$

The Hubble's parameter

$$H = \left(\frac{(2\,M - M^2)}{n}\right)\left(\frac{1}{t}\right) \tag{3.35}$$

$$\text{Where} H_x = \frac{3M}{nt} \; and \; H_y = H_z = \frac{3M(1-M)}{2(nt)} \tag{3.36}$$

Also

$$\frac{\sigma^2}{\theta^2} = \frac{(1+M)^2}{(2-M)^2} = \text{constant} \tag{3.37}$$

If $k = 0$, $n = 0$ and $A_1 = V^M$, where M is constant

$$\text{Since} \quad R = c_2 e^{lt} = V^{\frac{1}{3}} \tag{3.38}$$

$$A_1 = \left(c_2 e^{lt}\right)^{3M} \;, \qquad B_1 = \left(c_2 e^{lt}\right)^{\frac{3(M-1)}{2}} \tag{3.39}$$

Hence the metric (3.06) is

$$ds^2 = dt^2 - (e^t)^{3M} dx^2 - (e^t)^{\frac{3(M-1)}{2}}(dy^2 + dz^2) \tag{3.40}$$

with out loss of generality take $l=1$ $c_2 = 1$

The metric equation (3.40) represents an axially symmetric Bianchi type-I dark energy cosmological model for $k = 0$, $n = 0$

$$\rho = \frac{1}{8\pi G}\left[\left(\frac{9M(1-M)}{e^{2t}}\right) + \left(\frac{3(M-1)}{e^t}\right)^2 - \frac{(\phi_0{}^2\epsilon^2 - \phi_0\epsilon)}{t^2} + \left(\frac{3\phi_0\epsilon M}{te^t}\right) + \left(\frac{3\phi_0\epsilon M(M-1)}{te^t}\right) + \frac{3(\phi_0{}^2\epsilon^2)}{t^2}\right] \tag{3.41}$$

$$\boldsymbol{\omega} = \frac{-1}{8\pi G\rho}\left[\left[\frac{1}{2}\left(\frac{3(M-1)}{e^t}\right)^2 - \left(\frac{3(M-1)}{e^t}\right)\right] + \left(\frac{3(M-1)}{e^t}\right)^2 + \frac{\left(\phi_0{}^2\epsilon^2-\phi_0\epsilon\right)}{t^2} - \left(\frac{3\phi_0\epsilon M}{te^t}\right) + \left(\frac{3\phi_0\epsilon M(M-1)}{te^t}\right) - \frac{\left(\phi_0{}^2\epsilon^2\right)}{t^2}\right] \tag{3.42}$$

$$\gamma = \boldsymbol{\delta} = \frac{1}{8\pi G\rho}\left[\left[\frac{1}{2}\left(\frac{3\ (M-1)}{e^t}\right)^2 - \left(\frac{3\ (M-1)}{e^t}\right)\right] + \left(\frac{3\ (M-1)}{e^t}\right)^2 + \left(\frac{3\ \phi_0\epsilon M\ (M-1)}{te^t}\right) - \left(\frac{3\ M}{e^t}\right)^2 - \left(\frac{9\ M(1-M)}{e^{2t}}\right) - \left(\frac{3\ \phi_0\epsilon M}{te^t}\right)\right] \tag{3.43}$$

$$\phi = \phi_0 t^{\epsilon}, \qquad \epsilon = \pm 1, \pm\frac{1}{2} \tag{3.44}$$

$$V = (e^t)^3 \tag{3.45}$$

$$a = e^t \tag{3.46}$$

$$H = \left(\frac{2\ M-1}{e^t}\right) \tag{3.47}$$

$$\theta = \left(\frac{2\ M-1}{e^t}\right) \tag{3.48}$$

$$\sigma^2 = \frac{3}{4}\left(\frac{M+1)}{e^t}\right)^2 \tag{3.49}$$

$$A_l = \frac{-3M^2+10M-}{(2M-1)^2} \tag{3.50}$$

Also

$$\frac{\sigma^2}{\theta^2} = \frac{3}{4}\frac{(1+M)^2}{(2M-1)^2} = \text{constant} \tag{3.51}$$

***Case3*. If k > 0, $n > 1$, and $A_1 = V^M$, where M is constant**

$$\text{since, } R = c_3 e^{\frac{2}{n}\tanh^{-1}\left(\frac{kt}{n}-1\right)} = V^{1/3}$$

$$A_1 = \left(c_3 e^{\frac{2}{n}\tanh^{-1}\left(\frac{kt}{n}-1\right)}\right)^{3M}, \quad B_1 = \left(c_3 e^{\frac{2}{n}\tanh^{-1}\left(\frac{kt}{n}-1\right)}\right)^{\frac{3(M-1)}{2}} \tag{3.52}$$

Hence the metric (6) is

$$ds^2 = dt^2 - \left(e^{\frac{2}{n}\tanh^{-1}\left(\frac{kt}{n}-1\right)}\right)^{3M} dx^2 - \left(e^{\frac{2}{n}\tanh^{-1}\left(\frac{kt}{n}-1\right)}\right)^{\frac{3(M-1)}{2}} (dy^2 + dz^2) \tag{3.53}$$

with out loss of generality take $c_3 = 1$

The metric equation (3.53) represents an axially symmetric Bianchi type-I dark energy cosmological model if $k > 0,\ n > 1$.

$$\rho = \frac{1}{8\pi G}\left[\left(\frac{9M(M-1)n^4k^2}{2}\right)\left(\frac{1}{(k^2t-2kn)\tanh^{-1}\left(\frac{kt}{n}-1\right).}\right)^2 + \left(\frac{3(M-1)n^2k}{2(k^2t-2kn)\tanh^{-1}\left(\frac{kt}{n}-1\right).}\right)^2 - (\phi_0{}^2\epsilon^2 - \phi_0\epsilon) + \left(\frac{3Mn^2k\phi_0\epsilon}{2(k^2t-2kn)\tanh^{-1}\left(\frac{kt}{n}-1\right).}\right) + \left(\frac{3(M-1)n^2k\phi_0\epsilon}{(k^2t-2kn)\tanh^{-1}\left(\frac{kt}{n}-1\right).}\right) + 3(\phi_0{}^2\epsilon^2)\right]\left(\frac{1}{t^2}\right) \tag{3.54}$$

$$\boldsymbol{\omega} = -\frac{1}{8\pi G\rho}\left[\frac{3(M-1)k\left[n^2(2k^2t-2kn)\tanh^{-1}\left(\frac{kt}{n}-1\right)+2(k^2t^2-2knt)^2\right]}{\left((k^2t^2-2knt)\tanh^{-1}\left(\frac{kt}{n}-1\right)\right)^2.} + 3\left(\frac{3(M-1)n^2k}{2(k^2t^2-2knt)\tanh^{-1}\left(\frac{kt}{n}-1\right).}\right)^2 + \left(\frac{\phi_0{}^2\epsilon^2-\phi_0\epsilon}{t^2}\right) - \left(\frac{3Mn^2k\phi_0\epsilon}{2t(k^2t^2-2knt)\tanh^{-1}\left(\frac{kt}{n}-1\right).}\right) + 2\left(\frac{3(M-1)n^2k\phi_0\epsilon}{2t(k^2t^2-2knt)\tanh^{-1}\left(\frac{kt}{n}-1\right).}\right) - \left(\frac{\phi_0{}^2\epsilon^2}{t^2}\right)\right] \tag{3.55}$$

$$\boldsymbol{\gamma} = \boldsymbol{\delta} = \frac{-1}{8\pi G\rho}\left[\frac{3(M-1)k\left[n^2(2k^2t-2kn)\tanh^{-1}\left(\frac{kt}{n}-1\right)+2(k^2t^2-2knt)^2\right]}{\left((k^2t^2-2knt)\tanh^{-1}\left(\frac{kt}{n}-1\right)\right)^2.} + 2\left(\frac{3(M-1)n^2k}{2(k^2t^2-2knt)\tanh^{-1}\left(\frac{kt}{n}-1\right).}\right)^2 + 2\left(\frac{3(M-1)n^2k\phi_0\epsilon}{2t(k^2t^2-2knt)\tanh^{-1}\left(\frac{kt}{n}-1\right).}\right) - \frac{3Mk\left[n^2(2k^2t-2kn)\tanh^{-1}\left(\frac{kt}{n}-1\right)+2(k^2t^2-2knt)^2\right]}{\left((k^2t^2-2knt)\tanh^{-1}\left(\frac{kt}{n}-1\right)\right)^2.} - \left(\frac{3Mn^2k\phi_0\epsilon}{2t(k^2t^2-2knt)\tanh^{-1}\left(\frac{kt}{n}-1\right).}\right)\right] \tag{3.56}$$

$$\phi = \phi_0 t^{\epsilon}, \qquad \epsilon = \pm 1, \pm\frac{1}{2} \tag{3.57}$$

$$V = \left(c_3 e^{\frac{2}{n}\tanh^{-1}\left(\frac{kt}{n}-1\right)}\right)^{(6M-1)} \tag{3.58}$$

$$\theta = \left(\frac{3(3M-1)n^2k}{2(k^2t^2-2knt)\tanh^{-1}\left(\frac{kt}{n}-1\right).}\right) \tag{3.59}$$

$$\sigma^2 = 3\left(\frac{n^2k}{2(k^2t^2-2knt)\tanh^{-1}\left(\frac{kt}{n}-1\right).}\right)^2 \tag{3.60}$$

$$A_l = \frac{5-12M}{(3M-1)^2} \tag{3.61}$$

$$H = \left(\frac{(3M-1)n^2k}{2(k^2t^2-2knt)\tanh^{-1}\left(\frac{kt}{n}-1\right).}\right) \tag{3.62}$$

Also

$$\frac{\sigma^2}{\theta^2} = \frac{1}{3(3M-1)^2} = \text{constant} \tag{3.63}$$

In all three cases ,Clearly It can be observed that at early stage of universe the models (3.26, 3.40, 3.53) free from initial singularity. The physical quantities ρ, ω, γ , θ , σ^2, H are diverges and they become vanish for infinite value of t. The scalar field ϕ diverge when $\epsilon = 1, \frac{1}{2}$ while it vanishes when $\epsilon = -1, -\frac{1}{2}$. As t increases the spatial volume is also increases , which represents the universe is expanding. Since, A_l is constant the model is uniform throughout the evolution of the universe. Since, $\frac{\sigma^2}{\theta^2} = \text{constant}$, the model is anisotropy model at large values of t.

3.5.Conclusions

Dark energy cosmological models plays an prominent place in the general relativity which is observed for the accelerated expansion of the universe. The models developed with alternative theories of gravitation are obtaining more importance.. It is also observed that, the required model is anisotropic and free from initial singularity. All the physical parameters of this model are diverse at the initial epoch, $t = 0$ and tend to zero for large values of t.

CHAPTER-4

Anisotropic LRS Bianchi Type-II Dark Energy Model in Saez-Ballester Scalar-Tensor Theory of Gravitation.

4.1 Introduction

In recent years, there has been a lot of interest in dark energy models both in general relativity and in scalar-tensor theory of gravitation because of the fact that supernova 1a data (Perlmulter 1998, Reiss et al (1998)) and the observations of anisotropies in cosmic micro wave background radiation and the large scale structure have confirmed the accelerated expansion of the universe (Bennett et al 2003; verde et al 2002; Hawkins et al 2003; Abazajian et al. 2004). In chapter 1, a brief survey of scalar-tensor theories of gravitation and a discussion of the concept of dark energy is presented. It is well known that this expansion of the universe is driven by an exotic energy with large negative pressure which is known as dark energy. Inspite of an observational evidence dark energy is still a challenging problem in theoretical physics. To explain the cosmic positive acceleration, mysterious dark energy has been proposed. There are several dark energy models which can be distinguished by, for instance, their variable equation of state (EoS) $\omega(t) = \frac{p}{\rho}$. (where p is the fluid pressure and ρ its energy density) during the evolution of the universe. Methods allowing for restoration of the quantity $\omega(t)$ from expressional data have, been developed and an analysis of experimental data has been conducted to determine this parameter as a function of cosmic time. Sahni and Starobinsky 2006; Sahni et al 2008; Carrol and Haffman 2003; Ray et al.2010; Yadav et al 2011; Yadav and Yadav 2011; Pradhan and Amirhashchi 2011; Amirhashchi et al 2011; Pradhan et al 2010 are some of the authors who have investigated dark energy models with variable EoS parameter.

Spatially homogeneous and anisotropic cosmological models play significant role in the description of large scale behaviour of the universe and such models have been widely studied in the frame work of general relativity in search of a realistic picture of the universe in its early stages. Pradhan and Kumar (2009), Pradhan et al (2009, 11), Yadav et al.(2011), and Agarwal et al.(2011) have studied homogenous and an isotropic LRS Bianchi type-II space-time in different contexts. All of the above studies are based on the idea that an isotropic fluid give rise anisotropy in the expansion in LRS Bianchi type-II space-time. However, an anisotropic fluid must not necessarily promote the anisotropy in the expansion. Thus, even if we observe isotropic expansion in the present universe, we still cannot rule out the possibility of dark energy with an anisotropic equation of state (EoS) parameter. Rao et al.(2011) have obtained Bianchi type-I dark energy model in the scalar-tensor theory of Saez and Ballester(1986). Recently Pradhan and Amirhaschi (2011) obtained a new class of dark energy models in LRS Bianchi type-II space time with variable equation of state (EoS) parameter. Motivated by the above works, in this chapter, we have investigated a locally rotationally symmetric (LRS) Bianchi type-II space-time with variable equation of state (EoS) parameter and constant deceleration parameter in the scalar-tensor theory proposed by Saez and Ballester (1986).

The outline of this chapters is as follows: in section 4.2 the metric and field equations are described in Saez-Ballester scalar-tensor theory in the presence of anisotropic dark energy with prefect fluid energy momentum tensor. Section 4.3 deals with the solutions of the field equations with the help of a special law of variation of Hubble's parameter proposed by Bermann (1983), that yields constant deceleration parameter models of the universe. Section 4.4 is devoted to the discussion of some physical properties of the model obtained. 4.5 is Conclusions are summarized in the last section.

4.2 Metric and field equations

We consider a spatially homogeneous and anisotropic LRS Bianchi type-II space time given by

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + 2B^2xdydz + (B^2x^2 + A^2)dz^2 \qquad (4.01)$$

where A and B are functions of cosmic time only. By preserving the diagonal form of the energy momentum tensor in a consistent way with the above metric, the simplest generalization of EoS parameter of perfect fluid may be to determine it separately on each spatial axis. Therefore, the energy momentum tensor of perfect fluid is taken as

$$T_i^j = \text{diag}[T_0^0, T_1^1, T_2^2, T_3^3]\,. \qquad (4.02)$$

Thus, one may parameterize it as follows

$$\left.\begin{aligned} T_i^j &= \text{diag}\left[\rho, -p_x. -p_y, p_z\right] \\ &= \text{diag}\left[1, -\omega_x, -\omega_y, -\omega_z\right]\rho \\ &= \text{diag}[1, -(\omega+\delta), -\omega, -(\omega+\gamma)]\rho \end{aligned}\right\} \qquad (4.03)$$

where ρ is the energy density of the fluid, $p_x, p_y\ and\ p_z$ are the pressures and and ω_x , ω_y, ω_z are the directional EoS parameters along the x ,y and z axes respectively, ω is the deviation-free EoS parameter of the fluid. We have parameterized the deviation from isotropy by setting $\omega_y = \omega$ and then introducing skewness parameters δ and γ which are the deviations from ω along the x and z axes respectively. Also we obtain $\delta = \gamma$ because of the fact that in LRS Bianchi type-II space-time $T_1^1 = T_3^3$

The field equations given by Saez and Ballester (1986) for the combined scalar and tensor fields are

$$R_{ij} - \frac{1}{2}g_{ij}R - w\phi^n\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi'^{k}\right) = -T_{ij} \tag{4.04}$$

and the scalar field ϕ satisfies the equation

$$2\phi^n\phi^{i}_{;i} + n\phi^{n-1}\phi_{,k}\phi^{,k} = 0 \tag{4.05}$$

Here w and n are constants, T_{ij} is the energy momentum tensor of the matter and comma and semicolon denote partial and covariant differentiation respectively.

By adopting co moving coordinates the field equations (4.05) and (4.06) for the metric (4.01) with the help of equations (4.02) and (4.03) can be written as

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{1}{4}\frac{B^2}{A^4} - \frac{w}{2}\phi^n\dot{\phi}^2 = -(\omega + \gamma)\rho \tag{4.06}$$

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} - \frac{3}{4}\frac{B^2}{A^4} - \frac{w}{2}\phi^n\dot{\phi}^2 = -\omega\rho \tag{4.07}$$

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{A_4^2}{A^2} - \frac{1}{4}\frac{B^2}{A^4} + \frac{w}{2}\phi^n\dot{\phi}^2 = \rho \tag{4.08}$$

$$\ddot{\phi} + \dot{\phi}\left(2\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) + \frac{n}{2}\frac{\dot{\phi}^2}{\phi} = 0 \tag{4.09}$$

where an over head dot indicates differentiation with respect to t

4.4 Solutions of field equations

The field equations (4.06)-(4.09) are a system of four independent equations in six unknowns $A, B, \rho, \omega, \delta, \gamma$ and ϕ. Hence to find a determinate solution two more conditions are necessary. Firstly, we solve the system of non-linear equations with the help of special law of variation of Hubble's parameter proposed by Bermann (1983) that yields constant declaration parameter models of the universe.

The constant declaration parameter q is defined by

$$q = -\frac{a\ddot{a}}{(\dot{a})^2} = \text{constant} \qquad (410)$$

where the scale factor a for the space time(4.01) is given by

$$a = (A^2B)^{1/3} \qquad (4.11)$$

The solution of equation (4.11) is

$$a = (ct + d)^{1/1+q} \qquad (4.12)$$

Where $c \neq 0$ and d are constants of integration and $1 + q > 0$

The scalar of expansion θ, shear scalar $\sigma^2$are defined for the space time (4.01), by

$$\theta = 2\frac{\dot{A}}{A} + \frac{\dot{B}}{B} \qquad (4.13)$$

$$\sigma^2 = \frac{1}{3}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^2 \qquad (4.14)$$

Next we also assume that the expansion θ in the model is proportional to the shear scalar σ. This condition leads to

$$A = B^m \qquad (4.15)$$

where m is a constant. The motive behind this assumption is explained by Collins et al (1980)

Solving the field equations (4.06)-(4.09) with the help of (4.11),(4.12) and (4.15) we obtain the expressions for the metric coefficients as

$$A = (at + b)^{\frac{3m}{(2m+1)(1+q)}} \qquad (4.16)$$

$$B = (at + b)^{\frac{3}{(2m+1)(1+q)}} \qquad (4.17)$$

With a suitable choice of constants and the coordinates $(\mathrm{i.e.}\, c = 0, d = 1)$ the metric (4.01) with the help of (4.16) and (4.17) can be written as

$$ds^2 = -dt^2 + t^{\frac{6m}{(2m+1)(1+q)}}dX^2 + t^{\frac{6}{(2m+1)(1+q)}}dY^2 + 2t^{\frac{6}{(2m+1)(1+q)}}XdYdZ + \left[t^{\frac{6}{(2m+1)(1+q)}}X^2 + t^{\frac{6m}{(2m+1)(1+q)}}\right]dZ^2 \quad (4.18)$$

which represents LRS Bianchi type-II dark energy universe in Saez-Ballester theory.

4.4. Physical And Kinematical Properties of The Model

Eq(4.18) describes LRS Bianchi type –II cosmological model in the presence of dark energy in the scalar-tensor theory of gravitation proposed by Saez-Ballester(1986). With the following kinematical and physical parameters which are important for the discussion of the behavior of the model.

The spatial volume in the model is

$$V^3 = A^2B = t^{\frac{3m}{1+q}} \quad (4.19)$$

This shows that for accelerated expansion of the universe with time

$$1 + q > 0$$

The Hubble's parameter H, scalar expansion θ, shear scalar σ^2in the model are given by

$$H = \frac{3m}{(2m+1)(1+q)}\frac{1}{t} \quad (4.20)$$

$$\theta = \frac{9m}{(2m+1)(1+q)}\frac{1}{t} \quad (4.21)$$

$$\sigma^2 = \frac{3m^2}{(2m+1)^2(1+q)^2}\frac{1}{t^2} \quad (4.22)$$

From which we have

$$\frac{\sigma^2}{\theta^2} = \frac{1}{27} \neq 0 \tag{4.23}$$

From the above results one can observe that the parameters $H, \theta, \text{and } \sigma$ diverge at the initial epoch. Since $\frac{\sigma^2}{\theta^2} = \text{Constant}$ the model does not approach isotropy throughout the evolution of the universe

The physical parameters which are very important in the discussion of cosmology are:

Energy density

$$\rho = \frac{9m(m+1)}{(2m+1)^2(1+q)^2}\frac{1}{t^2} - \frac{1}{4t^{\frac{6(2m-1)}{(2m+1)(1+q)}}} + \frac{c_1 w}{2t^{\frac{1}{1+q}}} \tag{4.24}$$

EoS parameter of the fluid

$$\omega = \frac{1}{\rho}\left[\frac{6m}{(2m+1)(1+q)} - \frac{27m^2}{(2m+1)(1+q)^2}\right]\frac{1}{t^2} + \frac{3}{4}\frac{1}{t^{\frac{6(2m-1)}{(2m+1)(1+q)}}} + \frac{c_1 w}{2t^{\frac{6}{1+q}}} \tag{4.25}$$

Skewness parameter

$$\gamma = \delta = \frac{1}{\rho}\left[\left\{\frac{9(2m^2-m-1)^2}{(2m+1)^2(1+q)^2} - \frac{3(1-m)}{(2m+1)(1+q)}\right\}\frac{1}{t^2} - \frac{1}{t^{6(2m-1)}}\right] \tag{4.26}$$

And the scalar field is

$$\phi = \left[\frac{2}{n+2}\phi_0 t^{\frac{q-2}{1+q}}\right]^{\frac{2}{n+2}} \tag{4.27}$$

From eq (4.25) it may be observed that the EoS parameter ω is time dependent and hence it can also be function of scale factor as well. The parameters $\rho, \omega, \gamma = \delta$ diverge at initial epoch while they vanish for large values of t. The scalar field ϕ in the model has no initial singularity and approaches infinity for large t. Also, the model obtained is free from initial singularity, i.e at t$= 0$.

4.5 Conclusions

Construction of anisotropic dark energy models in general relativity is attracting the attention of more and more researchers because of the recent discovery of accelerated expansion of the universe. With the advent of modified theories, this problem has gained importance. Here we have investigated LRS Bianchi type-II anisotropic dark energy model in the scalar-tensor theory of gravitation formulated by Saez-Ballester (1986). Since scalar fields plays a significant role in the early stage of evolution of the universe, the model obtained and its properties throw a better light on our understanding of accelerated expansion of the universe.

CHAPTER-5

Spatially Homogeneous And Anisotropic Bianchi Type-III Dark Energy Model in f(R,T) Gravity

5.1 INTRODUCTION

Modified theories of gravity are attracting more and more attention of research works, in recent years, because of evidence of late time accelerated expansion of the universe which comes from high red shift supernova experiment(Riess et al.[1998,2004], Perlmutter et al.[1999], Bennett [2003]). Among the various modifications of general relativity, f(R) (Akbar and Cai [2006]) and f(R,T)(Harko et al. [2011]) theories of gravity are treated most seriously during the last decade. These theories are supposed to provide natural gravitational alternatives to dark energy. It has been suggested that cosmic acceleration can be achieved by replacing Einstein-Hilbert action of general relativity with a general function f(R) where R is a Ricci scalar. Chiba et al.[2007] ,Nojiri and Odintsov[2007,2011], Multamaki and Vilja [2006,2007], and Shamir [2010] are some of the authors who have investigated several aspects of f(R) gravity models which show the unification of early time inflation and late time acceleration.

Subsequently, Harko et al.[2011] proposed another extension of standard general relativity, f(R,T) modified theory of gravity wherein the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar R and of the trace of the stress-energy tensor T. They have derived the field equations from Hilbert-Einstein type variational principle. The covariant divergence of the stress energy tensor is also obtained. The f(R,T) gravity model depends on a source term, representing the variation of the matter energy tensor with respect to the metric. They have also demonstrated the possibility of reconstruction of arbitrary FRW cosmologies by an appropriate choice of a function f(T). A detailed discussion of f(R,T) gravity is presented in chapter-1.

Dark energy and dark energy models in general relativity and in modified theories of gravity have recently become an interesting subject of investigation for several authors (Sahni and Starobinsky [2000], Padmanabhan [2003], Caldwell [2002], Nojiri and Odintsov [2003], Kamenshchik et al. [2001] wang et al.[2005], Setare [2006], Naidu et al. [2012], Rao et al. [2012]). Dark energy is usually characterized by the equation of state(EoS) parameter given by $\omega(t) = \frac{p}{\rho}$ which is not necessarily constant, where p is the fluid pressure and ρ is energy density. The implication of the EoS parameter □ being function of cosmic time is to distinguish several dark energy models and to, conventionally, characterize the dark energy models. Caroll and Hoffman [2003], Ray et al. [2010], Akarsu and Kilinc [2010],Yadav and yadav [2011], Pradhan et al.[2011], Amirhashchi [2011] have studied dark energy models with variable EoS parameter.

Spatially homogeneous and anisotropic cosmological models are important in the discussion of large scale structure of the universe and such models have been studied in general relativity to realize the picture of the universe in it's early stages. Yadav et al. [2007], Pradhan et al. [2011] have investigated homogeneous and anisotropic Bianchi type-III space time in the context of massive strings. Yadav and Yadav [2010] have obtained Bianchi type-II anisotropic dark energy model with constant deceleration parameter. Recently, Prdhan and Amirhashchi [2011] have investigated a new anisotropic Bianchi type-III dark energy model, in general relativity, with equation of state (EoS) parameter without assuming constant deceleration parameter. Very recently, Naidu et al. [2012a,2012b,2012c] has presented Bianchi type-II and III dark energy models in Saez-Bellaster [1986] scalar-tensor theory of gravitation. While Reddy et al. [2012] has obtained five dimensional Kaluza-Klein cosmological model in f(R,T) gravity. Motivated by the above work's, we have investigated in this paper Bianchi-III anisotropic dark energy cosmological model with variable EoS parameter in f(R,T) gravity, choosing an appropriate form of f(T) proposed by Harko et al. [2011].

This chapter is organized as follows. In the section-5.2, the metric and the field equations are obtained in this theory. Section-5.3 deals with the solution of the field equations. In the section-5.4 some physical properties of the derived dark energy model are discussed. The last section 5.5 contains conclusions.

5.2.METRIC AND FIELD EQUATIONS

We consider spatially homogeneous and anisotropic Bianchi type-III metric given by

$$ds^2 = dt^2 - A^2(t)dx^2 - e^{-2\alpha x}B^2(t)dy^2 - C^2(t)dz^2 \qquad (5.01)$$

where A, B, and C are cosmic scale factors and m is a positive constant. The energy momentum tensor anisotropic dark energy is given by

$$T_j^i = diag\left\{\rho, -p_x, -p_y, -p_z\right\} = diag\{1, -\omega_x, -\omega_y, -\omega_z\}\rho \qquad (5.02)$$

where ρ is the energy density of the fluid and p_x, p_y, and p_z are the pressures along x, y, and z axes respectively. Here ω is the EoS parameter of the fluid and ω_x , ω_y, and ω_z are the EoS parameters in the directions of x, y, and z axes respectively. The energy momentum tensor can be parameterized as

$$T_j^i = diag[1, -\omega, -(\omega+\gamma), -(\omega+\delta)]\rho \qquad (5.03)$$

For the sake of simplicity we choose ω_x=ω and the skewness parameters γ and δ are the deviations from ω on y and z axes respectively.

In chapter 1, we have seen that the field equations of f(R,T) gravity can be written as (Harko et al.2011)

$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + \left(g_{ij}\Box - \nabla_i\nabla_j\right)f_R(R,T) = 8\pi T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\Theta_{ij} \qquad (5.04)$$

where

$$T_{ij} = -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g})}{\delta g^{ij}}L_m \ , \ \Theta_{ij} = -2T_{ij} - pg_{ij} \ , \ f_R(R,T) = \frac{\partial f(R,T)}{\partial R} \ and \ f_T(R,T) = \frac{\partial f(R,T)}{\partial T} \qquad (5.05)$$

where f(R,T) is an arbitrary function of Ricci scalar R and of the trace T of the stress energy tensor of matter T_{ij} *and* L_m is the matter lagrangian density and in the present study we have assumed that the stress energy tensor of matter as

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij} \tag{5.06}$$

Now assuming that the function f(R,T) given by Harko et al.(2011)

$$f(R,T) = R + 2f(T) \tag{5.07}$$

where f(T) is an arbitrary function of trace of the stress energy tensor of matter and

using Eqs. (5.05) and (5.06), the field equations (5.04) take the form

$$R_{ij} - \frac{1}{2}R g_{ij} = 8\pi T_{ij} + 2f'(T)T_{ij} + [2pf'(T) + f(T)]g_{ij} \tag{5.08}$$

where the overhead prime indicates differentiation with respect to the argument. We also choose

$$f(T) = \lambda T \tag{5.09}$$

where μ is constant [Harko et.al.(2011)].

Now assuming co-moving coordinate system, the field equations (5.08) for the metric (5.01) with the help of (5.02), (5.03) and (5.09) can be written as

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\frac{\dot{A}}{A} + \frac{\dot{C}}{C}\frac{\dot{B}}{B} - \frac{m^2}{A^2} = -\rho[8\pi + 2\lambda + 1 - 3\omega - \delta - \gamma] - 2\lambda p \tag{5.10}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{C}}{C}\frac{\dot{B}}{B} = \rho[(8\pi + 2\lambda)\omega - (1 - 3\omega - \delta - \gamma)] - 2\lambda p \tag{5.11}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{C}}{C}\frac{\dot{A}}{A} = \rho[(8\pi + 2\lambda)(\omega + \gamma) - (1 - 3\omega - \delta - \gamma)] - 2\lambda p \tag{5.12}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{B}}{B}\frac{\dot{A}}{A}-\frac{m^2}{A^2}=\rho[(8\pi+2\lambda)(\omega+\delta)-(1-3\omega-\delta-\gamma)]-2\lambda p \quad (5.13)$$

$$\frac{\dot{A}}{A}-\frac{\dot{B}}{B}=0 \quad (5.14)$$

where an overhead dot denotes differentiation with respect to t.

5.3. SOLUTION OF THE FIELD EQUATIONS

Integrating Eq.(5.14), we obtain

$$B=\alpha A \quad (5.15)$$

where α is constant of integration which can be taken as unity without loss of generality, so that we have

$$B=A \quad (5.16)$$

Using Eq.(8.16) in Eqs (5.11) and (5.13) we obtain

$$\gamma=0 \quad (5.17)$$

Now using Eqs. (5.16) and (5.17) in the field equations (5.10)-(5.13) we get

$$\left(\frac{\dot{A}}{A}\right)^2+2\frac{\dot{C}}{C}\frac{\dot{A}}{A}-\frac{m^2}{A^2}=-\rho[8\pi+2\lambda+1-3\omega-\delta]-2\lambda p \quad (5.18)$$

$$\frac{\ddot{C}}{C}+\frac{\ddot{A}}{A}+\frac{\dot{C}}{C}\frac{\dot{A}}{A}=\rho[(8\pi+2\lambda)\omega-(1-3\omega-\delta)]-2\lambda p \quad (5.19)$$

$$2\frac{\ddot{A}}{A}+\left(\frac{\dot{A}}{A}\right)^2-\frac{m^2}{A^2}=\rho[(8\pi+2\lambda)(\omega+\delta)-(1-3\omega-\delta)]-2\lambda p \quad (5.20)$$

The average scale factor and the spatial volume V are defined as

$$a=\sqrt[3]{A^2C} \quad , \quad V=a^3=A^2C \quad (5.21)$$

The general mean Hubbles's parameter H is given by

$$H=\frac{1}{3}(H_1+H_2+H_3) \quad (5.22)$$

where $H_1 = \frac{\dot{A}}{A} = H_2$, $H_3 = \frac{\dot{C}}{C}$ are the directional Hubble's parameters in the direction of x,y,z axes respectively. Using Eqs. (5.21) and (5.22) , we obtain

$$H = \frac{1}{3}(H_1 + H_2 + H_3) = \frac{\dot{a}}{a} \tag{5.23}$$

The expansion scalar θ and shear scalar σ are given by

$$\theta = 2\frac{\dot{A}}{A} + \frac{\dot{C}}{C} \tag{5.24}$$

$$\sigma^2 = \frac{1}{3}\left(\frac{\dot{A}}{A} - \frac{\dot{C}}{C}\right)^2 \tag{5.25}$$

The anisotropy parameter A_α is given by

$$A_\alpha = \sum\left(\frac{\Delta H_i}{H}\right)^2 \tag{5.26}$$

where $\Delta H_i = H_i - H$

The field equations (5.19)-(5.20) are three independent equations in six unknowns, A C , ρ, p, δ and ω. Hence to find a determinate solution three more conditions are necessary, we consider the following conditions :

(i) we apply the variation of Hubble's parameter proposed by Bermann(1983) that yields constant deceleration parameter models of the universe defined by

$$q = -\frac{a\ddot{a}}{\dot{a}^2}=\text{constant} \tag{5.27}$$

where the scale factor a is given by Eq. (5.22)

(ii) we assume that the scalar expansion θ is proportional to shear scalar σ which gives us (Collins et al.1980)

$$C = A^m \tag{5.28}$$

where m>1 is a constant.

(iii) The EoS parameter ω is propositional to skewness parameter δ (mathematical condition) such that

$$\omega + \delta = 0 \tag{5.29}$$

The solution of Eq. (5.27) is given by

$$a = (ct + d)^{\frac{1}{(1+q)}} \tag{5.30}$$

where $c \neq 0$ and d are constants of integration and $1+q>0$ for accelerated expansion of the universe.

Now using Eqs. (5.21), (5.28) and (5.30) the expressions for the metric coefficients in the field equations are

$$A = (ct + d)^{\frac{\varepsilon}{(m+z)(1+q)}} = B \tag{5.31}$$

$$C = (ct + d)^{\frac{\varepsilon m}{(m+z)(1+q)}} \tag{5.32}$$

Now with a suitable choice of coordinates and constants, the metric (5.01) with the help of (5.29) and (5.30) can be written as

$$ds^2 = dt^2 - t^{\frac{\varepsilon}{(m+z)(1+q)}}[dx^2 + e^{-m}\ dy^2] - t^{\frac{\varepsilon m}{(m+z)(1+q)}} dz^2$$

(5.33)

This model is similar to the Bianchi type-III dark energy model obtained by Pradhan and Amirhashchi (2011)

5.4 Physical And Kinematical Properties of The Model

Equation (5.33) represents Bianchi type-III dark energy model in f(R,T) gravity with the following physical and kinematical parameters of the model which are important for discussing the physics of the cosmological model.

The spatial volume in the model is

$$V = t^{\frac{2}{1+q}} \tag{5.34}$$

The general Hubble's parameter is

$$H = \frac{2}{3(1+q)t} \tag{5.35}$$

The scalar expansion in the model is

$$\theta = \frac{2}{(1+q)t} \tag{5.36}$$

The shear scalar in the model is

$$\sigma^2 = \frac{4(m-1)^2}{3(1+q)^2(m+2)^2t^2} \tag{5.37}$$

The mean anisotropy parameter is

$$A_\alpha = \frac{2(m-2)^2}{3(m+2)^2} \tag{5.38}$$

Now with the help of (5.29) we obtain the following physical parameters in the model (5.33)

The energy density in the model is

$$\rho = \frac{1}{(8\pi+2\lambda)}\left[\frac{8(1-m)-4(1+q)(m+2)}{(1+q)^2(m+2)^2}\right]\frac{1}{t^2} = -p \tag{5.39}$$

Since in the case of accelerated expansion we have $\rho + p = 0$.

The EoS and skewness parameters in the model are

$$\omega = -1 + \frac{1}{\rho(8\pi+2\lambda)}\left[\frac{8(1-m)-4(1+q)(m+2)}{(1+q)^2(m+2)^2}\right]\frac{1}{t^2} = -\delta \tag{5.40}$$

It may be observed that the cosmological model in f(R,T) gravity is free from initial singularity, i.e. at t=0. The spatial volume in this model increase as t increases confirming accelerated expansion of the universe. It can also be observed that $H, \theta, \sigma, \delta, \omega\ and\ p$ are functions of t and vanish for large t which they diverge for t=0.

Also, we have

$$\frac{\sigma^2}{\theta^2} = \frac{(m-1)^2}{3(m-2)^2} \neq 0 \tag{5.41}$$

and hence the model does not approach isotropy for large values of t. However the model becomes isotropy for m=1, because in this case, in view of Eqs. (5.16) and (5.28), the metric (5.01) becomes

$$ds^2 = dt^2 - A^2(t)[dx^2 - e^{-2\alpha x}dy^2 - dz^2] \tag{5.42}$$

It may be mentioned that the behavior of the physical parameters and the dark energy model in this case is quit similar to physical parameters in the Bianchi type –III dark energy model obtained by Prdhan and Amirhashchi (2011).

5.5. CONCLUSIONS

It is well known that anisotropy dark energy models with variable EoS parameter in modified theories of gravity play a vital role in the discussion of the accelerated expansion of the universe which is the crux of the problem in the present scenario. In this chapter we have investigated homogeneous and anisotropy Bianchi type-III dark energy model in f(R,T) gravity with variable EoS parameter in the perfect fluid source. It is observed that EoS parameter, skewness parameter in the model are all function of t. It can also be seen that the model is accelerating, expanding, non-rotating and has no initial singularity. This model confirms that high supernova experiment. The dark energy model obtained in this theory.

CHAPTER-6

Spatially Homogeneous and Bianchi type-V Dark Energy model in Saez-Ballester Scalar-Tensor Theory of Gravitation

6.1 Introduction

The observational evidence that our universe is making a transition from a decelerating phase to an accelerating phase is the major development in modern cosmology. This was confirmed by the observations of anisotropies in the cosmic microwave back ground (CMB) radiation as seen in the data from satellite such as WMAP and large scale structure (Bennett et al 2003, verde et al 2002, Spergel et al 2003). This has led to the construction of dark energy models with Variable EoS parameter. In chapter 1, a brief discussion of dark energy models is presented. Several authors such as Collins (1974), Coles and Ellis (1994), Maarten's and Nell (1978), Pradhan et al (2004), Yadav (2009) have studied isotropic dark energy models with EoS parameter in Bianchi type-V space-time in general relativity. Recently, Yadav (2011) has investigated some anisotropic dark energy models in Bianchi type-V space-time in Einstein's theory. Rao et al (2011) have studied LRS Bianchi type-I dark energy model with variable equation of state (EoS) in Saez-Ballester scalar-tensor theory of gravitation. We have obtained spatially homogeneous and anisotropic Bianchi type-II and Bianchi type–III dark energy models in Saez-Ballester (1986) scalar-tensor theory of gravitation.

Spatially homogeneous and anisotropic Bianchi type-V cosmological models create more interest as these models contain isotropic models as special cases and allow arbitrarily small anisotropy levels at any instant of cosmic time. This property makes them suitable as models of our universe. Also, Bianchi type-V models are more complicated and are simple generalization of FRW models with negative curvature. In this chapter, we investigate spatially homogeneous and anisotropic Bianchi type-V universe with variable equation of state (EoS) parameter and constant deceleration parameter in a scalar-tensor theory of gravitation proposed by Saez and Ballester (1986).

This chapter is organized as follows: in section 6.2, we obtain the field equations of Saez-Ballester scalar-tensor theory of gravitation with the aid of spatially homogeneous and anisotropic Bianchi type-V space-time in the presence of anisotropic dark energy with perfect fluid. Section 6.3 deals with the solutions of the field equations which will give us Bianchi type-V dark energy model in Saez-Ballester theory. Section 6.4 is devoted to the discussion of physical properties of the dark energy model. The last section 6.5 contains some conclusions.

6.2 Metric and Field Equations

We consider the spatially homogeneous and anisotropic Bianchi type-V space-time described by the line element

$$ds^2 = -dt^2 + A^2 dx^2 + e^{2\alpha x}(B^2 dy^2 + C^2 dz^2) \tag{6.01}$$

where A,B and C are functions of cosmic time t and α is a constant.

Saez and Ballester (1986) formulated a scalar-tensor theory of gravitation in which the metric is coupled with a dimensionless scalar field ϕ in a simple manner. The coupling gives a satisfactory description of the weak fields. Inspite of the dimensionless character of the scalar field an antigravity regime appears. This theory also suggests a possible way to solve missing matter problem in non-flat FRW cosmologies. The field equations for the combined scalar and tensor fields, in this theory, are

$$R_{ij} - \frac{1}{2} g_{ij} R - w\phi^n \left(\phi_{,i}\phi_{,j} - \frac{1}{2} g_{ij}\phi_{,k}\phi^{,k}\right) = -T_{ij} \tag{6.02}$$

and the scalar field $\emptyset$ satisfies the equation

$$2\phi^{n}\phi_{;i}^{,i} + n\phi^{n-1}\phi_{,k}\phi^{,k} = 0 \tag{6.03}$$

Also

$$T^{ij}_{;j} = 0 \tag{6.04}$$

is a consequence of the field equations (6.02) and (6.03). Here w and n are constants, T_{ij} is the energy momentum tensor of the fluid which is taken as

$$T_i^j = \text{diag}\,[T_0^0, T_1^1, T_2^2, T_3^3] \tag{6.05}$$

The simplest generalization of EoS parameter of perfect fluid is to determine it, separately, on each spatial axis by preserving diagonal form of the energy momentum tensor in a consistent way with the considered metric. Hence one can parameterized energy momentum tensor as follows

$$\left.\begin{aligned} T_i^j &= \text{diag}\,[\rho, -p_x, -p_y, -p_z] \\ &= \text{diag}\,[1, -\omega_x, -\omega_y, -\omega_z]\rho \\ &= \text{diag}\,[1, -(\omega+\delta), -\omega, -(\omega+\eta)]\rho \end{aligned}\right\} \tag{6.06}$$

Here ρ is the energy density of the fluid, $p_x, p_y,$ and p_z are the pressures and $\omega_x, \omega_y,$ and ω_z are the directional EoS parameters along the x, y, and z axes respectively, ω is the deviation free EoS parameter of the fluid. We parameterize the deviation from isotropy by setting $\omega_y = \omega$ and then introduce skewness parameters δ and η.

In co-moving coordinate system the independent field equations (6.02), and (6.03) for the metric (6.01) with the help of (6.06) can be written as

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{3\alpha^2}{A^2} + w\phi^{n}\dot{\phi}^2 = \rho \tag{6.07}$$

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}-\frac{\alpha^2}{A^2}-w\phi^n\dot{\phi}^2=-(\omega+\delta)\rho \tag{6.08}$$

$$\frac{\ddot{C}}{C}+\frac{\ddot{A}}{A}+\frac{\dot{C}\dot{A}}{CA}-\frac{\alpha^2}{A^2}-w\phi^n\dot{\phi}^2=-\omega\rho \tag{6.09}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{\alpha^2}{A^2}-w\phi^n\dot{\phi}^2=-(\omega+\eta)\rho \tag{6.10}$$

$$2\frac{\dot{A}}{A}-\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=0 \tag{6.11}$$

$$\ddot{\phi}+\dot{\phi}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)+\frac{n}{2}\frac{\dot{\phi}^2}{\phi}=0 \tag{6.12}$$

where an over head dot denotes ordinary differentiation with respect to t

6.3 Physical And Kinematical Properties of The Model

The field equations (6.07)-(6.12) are a system of six independent equations in eight unknown $A, B, C, \rho, \omega, \delta$ or $\gamma, \eta,$ and ϕ. Hence two additional constraints relating these parameters are required to obtain explicit solutions of the system.

Firstly we apply the variation of Hubble's parameter proposed by Bermann (1983) that yields constant deceleration parameter models of the universe. We consider only constant deceleration parameter model defined by

$$q=-\frac{a\ddot{a}}{(\dot{a})^2}=\text{Constant} \tag{6.13}$$

where the scale factor a is given by

$$a=(ABC)^{\frac{1}{3}} \tag{6.14}$$

Here the constant q is taken as negative (i.e. it is accelerating model of the universe)

The solution of equation (6.14) is

$$a = (ct + d)^{\frac{1}{1+q}} \tag{6.15}$$

0Where $c \neq 0$ and d are constants of integration. This equation implies that the condition of expansion is $1 + q > 0$.

Integrating equation (6.12) and absorbing the constant of integration in B or C, without loss of generality, we get

$$A^2 = BC \tag{6.16}$$

Using (6.16) in (6.14) and (6.15), we obtain

$$A = (at + b)^{\frac{1}{1+q}} \tag{6.17}$$

We also assume the relation between metric coefficients as

$$B = C^m \tag{5.18}$$

where m is a constant.

Now with the help of equations (6.16), (6.17) and (6.18) we can solve the field equations (6.07) – (6.12) and obtain the metric coefficients as follows

$$A = (ct + d)^{\frac{1}{1+q}} \tag{6.19}$$

$$B = (ct + d)^{\frac{2m}{(1+q)(1+m)}} \tag{6.20}$$

$$C = (ct + d)^{\frac{2}{(1+q)(1+m)}} \tag{6.21}$$

By a suitable choice of constants and coordinates ($i.e. c = 1, d = 0$) the metric (6.01) can, now, be written as

$$ds^2 = -dt^2 + t^{\frac{2}{1+q}}dX^2 + e^{2\alpha X}\left[t^{\frac{4m}{(1+q)(1+m)}}dY^2 + t^{\frac{4}{(1+q)(1+m)}}dZ^2\right] \tag{6.22}$$

which represents Bianchi type-V dark energy model in Saez-Ballester scalar-tensor theory of gravitation. The coordinate t, represents cosmic time.

6.4 Physical And Kinematical Properties of The Model

Equation (6.22) represents Bianchi type-V cosmological model with dark energy in Saez-Ballester theory. It may be observed that the model has no initial singularity. i.e. at $t = 0$

The physical and kinematical parameters in this model which are important for the discussion of cosmology are

Scalar field

$$\phi = \left[\frac{(n+2)}{2}\phi_0 t^{\frac{q-2}{1+q}}\right]^{\frac{2}{n+2}} \tag{6.23}$$

where ϕ_0 is a constant of integration

The energy density

$$\rho = \frac{2(m^2+4m+1)}{(1+q)^2(1+m)^2}\frac{1}{t^2} - \frac{3\alpha^2}{t^{\frac{2}{1+q}}} + \frac{c_1 w}{t^{\frac{6}{1+q}}} \tag{6.24}$$

EoS parameter

$$\omega = \frac{1}{\rho}\left\{\left[\frac{q(3+m)(1+m)-4}{(1+q)^2(1+m)^2}\right]\frac{1}{t^2} + \frac{\alpha^2}{t^{\frac{2}{1+q}}} + \frac{c_1 w}{t^{\frac{6}{1+q}}}\right\} \tag{6.25}$$

Skewness parameters δ and η

$$\delta = \frac{1}{\rho}\left[\frac{(1-m)(2-q)}{(1+q)^2(1+m)}\right]\frac{1}{t^2} \tag{6.26}$$

$$\eta = \frac{1}{\rho}\left[\frac{(1-m)(2-q-qm)}{(1+q)^2(1+m)^2}\right]\frac{1}{t^2} \tag{6.27}$$

The Spatial volume of the model

$$V^3 = ABC = t^{\frac{3}{1+q}} \tag{6.28}$$

Scalar of expansion

$$\theta = 3H = 3\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = \frac{9}{1+q}\frac{1}{t} \tag{6.29}$$

Shear scalar

$$\sigma^2 = \frac{1}{6}\theta^2 = \frac{27}{2}\frac{1}{(1+q)^2}\frac{1}{t^2} \tag{6.30}$$

Hubble's parameter

$$H = \frac{\theta}{3} = \frac{9}{2(1+q)^2}\frac{1}{t^2} \tag{6.31}$$

It is observed that from equation (6.28) for large t and $1 + q > 0$, spatial volume increases with time which indicates that the universe starts its expansion with zero volume from infinite past. Also for large values of t the scalar of expansionθ, shear scalar σ^2, Hubbles's parameter H tend to zero and they diverge at the initial epoch. The energy density, EoS parameterω , skewness parameter δ and η approach zero for large values of t and diverge for $t = 0$ while the scalar field ϕ increases for large values of t and$1 + q > 0$. Also $\frac{\sigma^2}{\theta^2} = \frac{3}{2} \neq 0$. Hence the model does not approach isotropy for large t.

6.5 Conclusions

The accelerated expansion of the universe can be studied using Bianchi type-I and V space-times which are the generalizations of FRW space-times. Also Scalar fields play a vital role in discussing dark energy models and early stages of evolution of the universe. Hence in this paper, we have studied Bianchi type-V dark energy model in the frame work of Seaz – Ballester (1986) scalar-tensor theory of gravitation. The model obtained is expanding and non-singular and does not approach isotropy for large t.

CHAPTER-7

High Dimensional Bianchi Type-III Dark Energy Cosmological Model in Scale Co-Variant Theory of Gravitation

7.1. Introduction

In recent years there has been a lot of interest in the study of alternative theories of gravitation(Brans and Dicke [1961] Nordtvedt [1970], Sen (1957) , Sen and Dunn (1971) and Seaz and Ballester(1985)). Canuto et al. (1977) formulated scale-covariant theory of gravitation which is a viable alternativeto general relativity (Wesson (1980); Will (1984)). In Brans-Dicke theory there existsa variable gravitational parameter G. In the scalecovariant theory Einstein's field equationsare valid in gravitational units where as physical quantities are measured in atomic units.The metric tensors in the two systems of units are related by a conformal transformation

$$\overline{g_{ij}} = \phi^2(x^k)g_{ij} \tag{7.01}$$

where in Latin indices take values 1, 2, 3, 4, bar denote gravitational units and unbar denotes

atomic quantities. The gauge function ϕ $(0 < \phi < \infty)$ in its most general formulation isfunction of all space-time coordinates. Thus using the conformal transformation of the typegiven by (7.01), Canuto et al. (1977) transformed the usual Einstein equations into

$$R_{ij} - \frac{1}{2}Rg_{ij} + f_{ij}(\phi) = -8\pi G(\phi)T_{ij} + \Lambda(\phi)g_{ij} \tag{7.02}$$

where

$$\phi^2 f_{ij} = 2\phi\phi_{i;j} - 4\phi_i\phi_j - g_{ij}(\phi\phi'^k_{ik} - \phi^k\phi_k) \tag{7.03}$$

Here $R_{ij}$is the Ricci tensor, R is the Ricci scalar, $\Lambda$is the cosmological 'constant', G is the

gravitational 'constant' and $T_{ij}$is the energy momentum tensor. A semi colon denotes covariantderivative and $\phi_i$denotes ordinary derivative with respect to x^i.A particular featureof this theory is that no independent equation for ϕexists. The possibilities that have been

considered for gauge function φ (Canuto et al. (1977)) are

$\phi(t) = \left(\frac{t_0}{t}\right)^{\varepsilon}, \varepsilon = \pm 1, \pm \frac{1}{2}$(4)

where$t_0$is a constant. The form

$$\phi \sim t^{\frac{1}{2}} \tag{7.04}$$

is the one most favored to fit observations (Canuto and Goldman (1983)). Reddy and Rao (2001) ,Reddy et al. (2002) , Reddy and Naidu (2007), Beesham (1986a,1986b,1986c), Reddy and Venkateshwarlu (1987,2004) and Singh and Devi (2011) are some of the authors who have investigated several aspects of the scale covariant theory of gravitation. The discovery of the accelerated expansion of the universe supposedly driven by exoticdark energy (Perlmutter et al. (1999), Reiss et al. (1998), Spergel et al. (2003,2007), Copeland et al (2006)) has lead, in recent years, to the investigation of dark energy models both in general relativity and in alternative theories of gravitation. The nature and composition of dark energyis still an open problem. Dark energy is usually characterized by the EoS parameter$\omega(t) = \frac{p}{\rho}$which is not necessarily constant, where p is the fluid pressure and ρ is the energydensity (Carroll and Hoffman (2003)). A lucid introduction and nice review of the work doneon dark energy model in general relativity is given by Farooq et al. (2011) and Pradhanetal. (2011) Pradhan and Amihaschi (2011), 28], Amirhashchi et al. [29], Pradhan et al. [30] havediscussed dark energy models in anisotropic Bianchi type space times with variable EOSparameter. Recently Naidu et al. (2011) have obtained Bianchi type-II and V dark energymodels in the scalar-tensor theory of gravitation proposed by Saez and Ballester (1985). Spatially homogeneous and anisotropic cosmological models play a vital role in the study of the early stages of evolution of the universe. Here we have investigated Higher order Bianchi type-III dark energy model in the scale-covariant theory of gravitation formulated by Canuto et al (1971).

7.2 Metric and Field Equations

We consider a spatially homogeneous higher order Bianchi type-III metric in the form

$$ds^2 = dt^2 - A^2(t)dx^2 - e^{-2\alpha x}B^2(t)dy^2 - C^2(t)dz^2 - D^2(t)du^2 \tag{7.06}$$

where$A$,B,C *and* D are functions of cosmic time t only.

One may determine EoS parameter of perfect fluid separately on each spatial axis bypreserving the diagonal form of the energy momentum tensor in a consistent way with the metric (7.06).Therefore, the energy momentum tensor of the fluid is taken as

$$T_j^i = diag\ \{T_0^0,\}T_1^1, T_2^2, T_3^3, T_4^4 \tag{7.07}$$

We can parameterize as follows

$$\begin{aligned} T_j^i \quad &= diag\ \{\rho, -p_x, -p_y, -p_z, -p_u\} \\ &= diag\{1, -\omega_x, -\omega_y, -\omega_z, -\omega_u\} \\ &= diag\{1, -\omega, -(\omega+\gamma) - (\omega+\delta), -(\omega+\eta)\}\rho \end{aligned} \tag{7.08}$$

whereρ is the energy density of the fluid, $p_x, p_y, p_z, and\ p_u$are the pressures and $\omega_x, \omega_y, \omega_z, \omega_u$are thedirectional EoS parameters along the *x,y,z and u* -axes respectively. $\omega$is the deviation freeEoSparameter of the fluid. We have parameterized the deviation from isotropy by setting $\omega_x = \omega$ and then introducing skewnessparameters$\delta, \gamma$and η, that is deviation from ωalong they, zand $u$axes respectively.

In a comoving coordinate system the field equations (7.02) and (7.03) of the scale-covariant theory

for the metric (7.06) with help of (7.07) and (7.08) take the form.

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\frac{\dot{A}}{A} + \frac{\dot{A}}{A}\frac{\dot{D}}{D} + \frac{\dot{C}}{C}\frac{\dot{B}}{B} + \frac{\dot{B}}{B}\frac{\dot{D}}{D} + \frac{C}{C}\frac{\dot{D}}{D} + \left[\frac{\dot{A}}{A} + \frac{B}{B} + \frac{\dot{C}}{C} + \frac{\dot{D}}{D}\right]\left(\frac{\dot{\phi}}{\phi}\right) - \frac{\alpha^2}{A^2} - \frac{\ddot{\phi}}{\phi} + 4\left[\frac{\dot{\phi}}{\phi}\right]^2 = 8\pi G \tag{7.09}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + + \frac{\ddot{D}}{D} + \frac{\dot{C}}{C}\frac{\dot{B}}{B} + \frac{\dot{B}}{B}\frac{\dot{D}}{D} + \frac{C}{C}\frac{\dot{D}}{D} + \left[\frac{\dot{A}}{A} + \frac{B}{B} + \frac{\dot{C}}{C} + \frac{\dot{D}}{D}\right]\left(\frac{\dot{\phi}}{\phi}\right) + \frac{\ddot{\phi}}{\phi} - 2\left(\frac{\dot{A}\dot{\phi}}{A\phi}\right) = 8\pi G(\omega)\rho \tag{7.10}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\ddot{D}}{D}+\frac{\dot{A}}{A}\frac{\dot{C}}{C}+\frac{\dot{A}}{A}\frac{\dot{D}}{D}+\frac{\dot{C}}{C}\frac{\dot{D}}{D}+\left[\frac{\dot{A}}{A}+\frac{B}{B}+\frac{\dot{C}}{C}+\frac{\dot{D}}{D}\right]\left(\frac{\dot{\phi}}{\phi}\right)+\frac{\ddot{\phi}}{\phi}-2\left(\frac{\dot{B\phi}}{B\phi}\right)=8\pi G(\omega+\gamma)\rho \quad (7.11)$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\ddot{D}}{D}+\frac{\dot{A}}{A}\frac{\dot{B}}{B}+\frac{\dot{A}}{A}\frac{\dot{D}}{D}++\frac{\dot{B}}{B}\frac{\dot{D}}{D}+\left[\frac{\dot{A}}{A}+\frac{B}{B}+\frac{\dot{C}}{C}+\frac{\dot{D}}{D}\right]\left(\frac{\dot{\phi}}{\phi}\right)-\frac{\alpha^2}{A^2}+\frac{\ddot{\phi}}{\phi}-2\left(\frac{\dot{C\phi}}{C\phi}\right)=8\pi G\ (\omega+\delta)\rho$$

(7.12)

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{A}}{A}\frac{\dot{B}}{B}+\frac{\dot{A}}{A}\frac{\dot{C}}{C}++\frac{\dot{B}}{B}\frac{\dot{C}}{C}+\left[\frac{\dot{A}}{A}+\frac{B}{B}+\frac{\dot{C}}{C}+\frac{\dot{D}}{D}\right]\left(\frac{\dot{\phi}}{\phi}\right)-\frac{\alpha^2}{A^2}+\frac{\ddot{\phi}}{\phi}-2\left(\frac{\dot{D\phi}}{D\phi}\right)=8\pi G\ (\omega+\eta)$$

(7.13)

$$\frac{\dot{A}}{A}-\frac{\dot{B}}{B}=0 \quad (7.14)$$

where an overhead dot denotes ordinary differentiation with respect to t .

7.3 SOLUTION OF THE FIELD EQUATIONS

Integrating Eq.(7.14), we obtain

$$B=kA \quad (7.15)$$

where kis constant of integration which can be taken as unity without loss of generality,so that we have

$$B=A \quad (7.16)$$

Using Eq.(7.16) in Eqs 7.(10) and (7.11) we obtain

$$\gamma=0 \quad (7.17)$$

Now using Eqs. (7.16) and (7.17) in the field equations (7.09)-(7.14) we get

$$\left(\frac{\dot{A}}{A}\right)^2+2\frac{\dot{C}}{C}\frac{\dot{A}}{A}+2\frac{\dot{A}}{A}\frac{\dot{D}}{D}+\frac{C}{C}\frac{\dot{D}}{D}+\left[2\frac{\dot{A}}{A}++\frac{\dot{C}}{C}+\frac{\dot{D}}{D}\right]\left(\frac{\dot{\phi}}{\phi}\right)-\frac{\alpha^2}{A^2}-\frac{\ddot{\phi}}{\phi}+4\left[\frac{\dot{\phi}}{\phi}\right]^2=8\pi G\rho \quad (7.18)$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\ddot{D}}{D}+\frac{\dot{C}}{C}\frac{\dot{A}}{A}+\frac{\dot{A}}{A}\frac{\dot{D}}{D}+\frac{C}{C}\frac{\dot{D}}{D}+\left[2\frac{\dot{A}}{A}+\frac{\dot{C}}{C}+\frac{\dot{D}}{D}\right]\left(\frac{\dot{\phi}}{\phi}\right)+\frac{\ddot{\phi}}{\phi}-2\left(\frac{\dot{A\phi}}{A\phi}\right)=8\pi G(\omega)\rho \quad (7.19)$$

$$2\frac{\ddot{A}}{A}+\frac{\ddot{D}}{D}+\left(\frac{\dot{A}}{A}\right)^2+2\frac{\dot{A}}{A}\frac{\dot{D}}{D}+\left[2\frac{\dot{A}}{A}+\frac{\dot{C}}{C}+\frac{\dot{D}}{D}\right]\left(\frac{\dot{\phi}}{\phi}\right)-\frac{\alpha^2}{A^2}+\frac{\ddot{\phi}}{\phi}-2\left(\frac{\dot{C\phi}}{C\phi}\right)=8\pi G\ (\omega+\delta)\rho \quad (7.20)$$

$$2\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\left(\frac{\dot{A}}{A}\right)^2+2\frac{\dot{A}}{A}\frac{\dot{C}}{C}+\left[2\frac{\dot{A}}{A}+\frac{\dot{C}}{C}+\frac{\dot{D}}{D}\right]\left(\frac{\dot{\phi}}{\phi}\right)-\frac{\alpha^2}{A^2}+\frac{\ddot{\phi}}{\phi}-2\left(\frac{D\dot{\phi}}{D\phi}\right)=8\pi G\,(\omega+\eta)\rho \quad (7.21)$$

The spatial volume for the metric (6) is given by

$$V = A^2CD \quad (7.22)$$

We define the average scale factor of the metric (6) as

$$R = \sqrt[3]{A^2CD} \quad (7.23)$$

The general mean Hubbles's parameter H is given by

$$H = \frac{1}{3}(H_x + H_y + H_z + H_u) \quad (7.24)$$

where$H_x = \frac{\dot{A}}{A} = H_y$, $H_z = \frac{\dot{C}}{C}$, $H_u = \frac{\dot{D}}{D}$ are the directional Hubble's parameters in the direction of x,y,zaxes respectively.

Using Eqs. (7.22),(7.23) and (7.24) ,

we obtain

$$H = \frac{1}{3}\left(H_x + H_y + H_z + H_u\right) = \frac{\dot{R}}{R} \quad (7.25)$$

The scalar expansion θ is given by

$$\theta = 2\frac{\dot{A}}{A} + \frac{\dot{C}}{C} + \frac{\dot{D}}{D} \quad (7.26)$$

The shear scalar σ

$$\sigma^2 = \frac{1}{2}\left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2 + \left(\frac{\dot{D}}{D}\right)^2 - \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\frac{\dot{A}}{A} - \frac{\dot{A}}{A}\frac{\dot{D}}{D} - \frac{\dot{C}}{C}\frac{\dot{B}}{B} - \frac{\dot{B}}{B}\frac{\dot{D}}{D} - \frac{\dot{C}}{C}\frac{\dot{D}}{D}\right] \quad (7.27)$$

The anisotropy parameter A_α is given by

$$A_\alpha = \sum\left(\frac{\Delta H_i}{H}\right)^2 \quad (7.28)$$

where$\Delta H_i = H_i - H$ (i=1,2,3,4)

The field equations (9)-(14) are three independent equations in eight unknowns, A C,D , ρ, p, δ, η and ω. Hence to find a determinate solution three more conditions are necessary, we consider the following conditions :

(iv) we apply the variation of Hubble's parameter proposed by Bermann(1983) that yields constant deceleration parameter models of the universe defined by

$$q = -\frac{R\ddot{R}}{\dot{R}^2} = \text{constant} \tag{7.29}$$

where the scale factor R is given by Eq. (7.22) and (7.23)

(v) we assume that the scalar expansion θ is proportional to shear scalar σ which gives us (Collins et al.1980)

$$C = A^m \qquad \text{and} \qquad D = A^n \tag{7.30}$$

where m>1 and n>1 are constants.

(vi) The EoS parameter ω is propositional to skewness parameters δ and η(mathematical condition) such that

$$\omega + \delta = 0 \qquad\qquad \omega + \eta = 0 \tag{7.31}$$

The solution of Eq. (29) is given by

$$R = (ct + d)^{\frac{1}{(1+q)}} \tag{7.32}$$

where c≠0 and d are constants of integration and 1+q>0 for accelerated expansion of the universe.

Now using Eqs. (22)-(23), (30) and (32) the expressions for the metric coefficients in the field equations are

$$A = (ct + d)^{\frac{3}{(m+n+2)(1+q)}} = B \tag{7.33}$$

$$C = (ct + d)^{\frac{3m}{(m+n+2)(1+q)}} \qquad D = (ct + d)^{\frac{3n}{(m+n+2)(1+q)}} \tag{7.34}$$

Now with a suitable choice of coordinates and constants, the metric (7.01) with the help of (7.31) -(7.32) can be written as

$$ds^2 = dt^2 - t^{\frac{3}{(m+n+2)(1+q)}}[dx^2 + e^{-mx}dy^2] - t^{\frac{3m}{(m+n+2)(1+q)}}dz^2 - t^{\frac{3n}{(m+n+2)(1+q)}}du^2 \quad (7.35)$$

This model represents A higher dimensionalBianchi type-III dark energy model.

7.4 Physical And Kinematical Properties of The Model

The metric given by (7.35) represents higher dimensionalBianchi type-III dark energy cosmologicalmodel with the following physical and kinematical parameters in scale co-variant theory of gravitation .

The energy density

$$\rho = \frac{1}{8\pi G}[1 + 2m + 2n + 2mn + (2 + m + n)\phi_0\varepsilon - \phi_0\varepsilon(\varepsilon - 1) + 4(\phi_0\varepsilon)^2]\left[\frac{1}{t^2}\right] - \alpha^2\left[\frac{1}{t}\right]^{\frac{6}{(m+n+2)(1+q)}} = -p \quad (7.36)$$

since accelerated expansion of the universe $\rho + p = 0$

EoS parameter

$$\omega = \frac{1}{8\pi G\rho}\left[\frac{3(1+m+n)}{(m+n+2)(q+1)}\left(\frac{1}{t}\right) + \frac{9(m+n+mn)}{(m+n+2)^2(q+1)^2}\left(\frac{1}{t^2}\right)\right]$$

$$+\left[(2 + m + n)\phi_0\varepsilon\left(\frac{1}{t^2}\right) - 2\phi_0\varepsilon\left(\frac{3}{(m+n+2)(q+1)}\right)\left(\frac{1}{t}\right)^2\right] \quad (7.37)$$

Skewness parameters

$$\delta = \eta = -\omega = \frac{-1}{8\pi G\rho}\left[\frac{3(1+m+n)}{(m+n+2)(q+1)}\left(\frac{1}{t}\right) + \frac{9(m+n+m\ \ )}{(m+n+2)^2(q+1)^2}\left(\frac{1}{t^2}\right)\right]$$

$$-\left[(2 + m + n)\phi_0\varepsilon\left(\frac{1}{t^2}\right) - 2\phi_0\varepsilon\left(\frac{3}{(m+n+2)(q+1)}\right)\left(\frac{1}{t}\right)^2\right] \quad (7.38)$$

Scalar field

$$\phi = \phi_0 t^n \quad , \quad \varepsilon = \pm 1, \pm\frac{1}{2} \quad (7.39)$$

The spatial volume

$$V^3 = t^{\frac{3}{(1+q)}} \tag{7.40}$$

Scalar expansion

$$\theta = \frac{3}{(1+q)t} \tag{7.41}$$

Shear scalar

$$\sigma^2 = \frac{9}{2}\left[\frac{(m+n-1)^2+mn}{(m+n+2)^2(1+q)^2}\right]\left[\frac{1}{t^2}\right] \tag{7.42}$$

Average anisotropy parameter

$$A_m = \frac{[2-m-n]}{3[2+m+n]} \tag{7.43}$$

Hubble's parameter

$$H = \left(\frac{1}{1+q}\right)\left(\frac{1}{t}\right) \tag{7.44}$$

Also$\frac{\sigma^2}{\theta^2} = \left[\frac{(m+n-1)^2+mn}{(m+n+2)^2}\right]$ = constant (7.45)

It can be observed that the model (7.35) has no initial singularity, i.e. at $t = 0$. Physical quantities,ρ, ω, γ diverge at $t = 0$ while they vanish for large values of t . The scalar field ϕtendsto infinity for large t when $\varepsilon = 1, \frac{1}{2}$, while it vanishes when $\varepsilon=-1, -\frac{1}{2}$. The spatial volume increases as t increases (since $1 + q > 0$) which shows that the universe is expanding. The scalar of expansion θ, shear scalar $\sigma^2$and the Hubble's parameter H diverge at $t = 0$ and vanish for large t . The mean anisotropic parameter is uniform throughout the evolution of the universe, since it does not depend on the cosmic time t. Since $\frac{\sigma^2}{\theta^2}$= *constant*, the model does not approach isotropy for large values of t .However, since $1 + q > 0$ the universe is expanding with the increase of cosmic time but the rate of expansion and shear scalar decrease to zero and tend to isotropic. Thus this case implies an accelerating model of universe. Hence it follows that our model represents physical dark energy model.

7.5 Conclusions

Dark energy cosmological models are, recently, playing a vital role in the discussion of accelerated expansion of the universe in general relativity. With the advent of alternative theories of gravitation study of these models is gaining importance. Here we have investigate higher order Bianchi type-III dark energy model with variable EoS parameter in a scale-covariant theory of gravitation formulated by Canuto et al. [1977]. It is observed that the model has no initial singularity and all the physical parameters are infiniteat the initial epoch, $t = 0$ and tend to zero for large t . It is also observed that the model does not approach isotropy through the whole evolution of the universe. We hope that the model is useful for understanding of dark energy model in scale covariant theory of gravitation.

Summery And Final Conclusion of the Report

Summery of the Report:

In view of the fact that with Bianchi type dark energy cosmological models have gained importance in recent years, we have discussed in chapter A general class of Bianchi Cosmological Model in the presence of a perfect fluid and non rotating dark energy is considered. A determinate solution is obtained with a special law of variation for Hubble's parameter propose by Bermann(1983) is used in a general class of Bianchi Cosmological Model in the presence of a perfect fluid and dark energy, hence non-rotaing model is obtained. An anisotropic dark energy parameter in the spatially homogeneous Bianchi-type-I with Equation of State (EoS) parameter in the frame work of Scale covariant theory is obtained . This model will help to study the role of dark energy in getting accelerated expansion of the universe popularly known as inflationary phase. Spatially homogeneous and anisotropic cosmological model play significant role in the description of large scale behavior of the universe and realistic picture of the universe in its early stages.So, a spatially homogeneous and anisotropic LRS Bianchi type-II space-time dark energy is considered in sacalar-tensor theory of gravitation proposed by Saez-Ballester(1986) obtained . The discussion of a dark energy cosmological model with Equation of State (EoS) parameter in f(R,T) gravity in Bianchi type-III space-time in the presence of a perfect fluid source. the plausible physical conditions that the scalar expansion is proposed to the shear scalar (Collins et al. 1980) and the EoS parameter is proportional to skewness parameter. It is observed that the EoS parameter and the skewness parameter in the model turn out to be functions of cosmic time which will help in discussing late - time acceleration [Reiss et al.(1998,2004)]. Spatially homogeneous and Bianchi type-V cosmological model create interest in isotropic models as special cases and allow arbitrarily small anisotropy levels at any instant of cosmic time. The model obtained High dimensional dark energy cosmological model with Equation of State (EoS) parameter in scale-covariant theory of gravitation formulated by Canuto et al.(1977) in presence of perfect fluid will also help to study the structure formation of the universe in early stage and will definitely help in the discussion of accelerated expansion of the universe with special reference to scale covariant theory of gravitation. The physical and kinematical properties and their behaviors of the all above said model are discussed.

Final Conclusion of the Report:

We came to observe here that

1. Anisotropy dark energy models with variable EoS parameter play a prominent role in the discussion of the accelerated expansion of the universe , the crux of the problem in the present scenario.
2. The homogeneous and anisotropy General class of Bianchitype-Type dark energy model with variable EoS parameter have been investigated. It is identified that EoS parameter, skewness parameter in the model are all function of t. It can also be seen that the model is accelerating, expanding, non-rotating and free from initial singularity. Also, this model confirms that high supernova experiment.
3. The Models obtained are anisotropic and free from initial singularity.
4. Since scalar fields plays a significant role in the early stage of evolution of the universe, the models obtained and its properties throw a better light on our understanding of accelerated expansion of the universe.
5. It is observed that EoS parameter, skewness parameter in the model are all function of t.

6. It can also be seen that the model is accelerating, expanding, non-rotating and has no initial singularity.
7. The dark energy model obtained in this theory are model confirms that high supernova experiment.
8. we have investigate higher order Bianchi type-III dark energy model is obtained with variable EoS parameter in a scale-covariant theory of gravitation formulated by Canuto et al. [1977].
9. It is observed that the model obtained has no initial singularity and all the physical parameters are infinite at the initial epoch, $t = 0$ and tend to zero for large t .
10. It is also observed that the model does not approach isotropy through the whole evolution of the universe.
11. All the physical parameters of this model are diverse at the initial epoch, $t = 0$ and tend to zero for large values of t.

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"General Bianchi Type Dark Energy Cosmological model in Modified theories of Gravitation"

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General Bianchi Type Dark Energy Cosmological model in Modified theories of Gravitation

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ABSTRACT. The general class of a dark energy Bianchi model has been presented in presence of perfect fluid. The Characteristics of the model discussed.

Keywords: Dark Energy, EoS Parameter, Bianchi type models,

I. INTRODUCTION

Alternative theories of gravity draw high attention of research works, in recent past years, by studying the strong proof of late time accelerated expansion of the universe, which occurred from high red shift supernova experiment(ref. 1-4). The general class of Bianchi-Type models will introduce the gravitational alternatives to dark energy.

The alternative theories of gravitation with the dark energy models are very prominent in In general relativity to make the great research of the several authors (ref.5-6). In Dark energy , the equation of state parameter i.e., $\omega(t) = \frac{p}{\rho}$ is a not necessary constant so function of cosmic time t , where p and ρ are pressure and density of the model respectively. To because $\omega(t) = \frac{p}{\rho}$ is not a constant function hence it is characterize the dark energy models conventionally as well as distinguish the cosmological models. Several Dark energy models have made by several authors(ref 7-8).

Anisotropic and Spatially homogeneous cosmological models have become very prominent in the research of large scale structure of the universe. These models also studied to understand the actual situation happened at the time of structure formation of the universe. Researcher (ref.9) have researched scholarly anisotropic and homogeneous and Bianchi type-III space time in the context of massive strings. The author (ref.10) has investigated anisotropic Bianchi type-III dark energy model. The authors (ref.11-12) have obtained Bianchi type-II and III dark energy models in Saez-Bellaster scalar-tensor with f(R,T) theory of gravitation. The author (ref 13) has presented general class of Bianchi type model with f(R,T) theory of gravitation.

The inspiration of above researches , in this paper I developed a General class of Bianchi-Type anisotropic dark energy cosmological model with variable EoS parameter .

This paper covers the following sections. (II) Field equations of the metric. (III) Solution of the field equations. (IV) Some Characteristics and physical behavior of the model. (V) Conclusions.

II. METRIC AND FIELD EQUATIONS

The general class of Bianchi Cosmological model in diagonal form is

$$\boldsymbol{ds^2 = dt^2 - A^2(t)dx^2 - e^{-2x}B^2(t)dy^2 - C^2e^{-2mx}(t)dz^2} \quad (1)$$

Here A, B, and C are cosmic scale factors , m (>0)is constant.

The metric (1) represents, Bianchi type-III model (If m=0) , Bianchi type-VI_0 (If m= -1) , Bianchi type-V (if m= 1), Bianchi type-VI_h model (If m=h-1) .

The Einstein field equations in case of dark energy components take the form

$$G_{ij} = R_{ij} - \frac{1}{2} R g_{ij} = -T_{ij} \tag{2}$$

Where $8\pi G = 1, c = 1$

The combined energy momentum tensor (for dark energy and perfect fluid) is

$$T_j^i = (\rho_{tot} + p_{tot}) u_i u_j - p_{tot}\, g_{ji} = (\rho + p) u_i u_j - p g_{ji} \tag{3}$$

where u^i co-moving vectors such that $u^i = (0,0,0,1) and\ u_i u^j = 1$

The non-vanishing components of energy momentum tensor are ,

$$T_1^1 = T_2^2 = T_3^3 = -p_{tot}\ and\ T_4^4 = \rho_{tot} \tag{4}$$

Here $\rho = \rho_{tot} = \rho_{DE} + \rho_{PF}$, $p = p_{tot} = p_{DE} + p_{PF}$

Where ρ is energy density , p is pressure of the fluid

The equation of state parameter of the fluid is

$$\omega = \frac{p}{\rho} \tag{5}$$

The field equations for the metric (1) by Eqs. (2), (3), (4) and (5), Using co-moving coordinate system ,we obtain

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\frac{\dot{A}}{A} + \frac{\dot{C}}{C}\frac{\dot{B}}{B} - \frac{m^2+m+1}{A^2} = \rho_{DE} + \rho_{PF} = \rho \tag{6}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{C}}{C}\frac{\dot{B}}{B} - \frac{m}{A^2} = -(p_{DE} + p_{PF}) = -p \tag{7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{C}}{C}\frac{\dot{A}}{A} - \frac{m^2}{A^2} = -(p_{DE} + p_{PF}) = -p \tag{8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{B}}{B}\frac{\dot{A}}{A} - \frac{1}{A^2} = -(p_{DE} + p_{PF}) = -p \tag{9}$$

$$(m+1)\frac{\dot{A}}{A} - \frac{\dot{B}}{B} - m\frac{\dot{C}}{C} = 0 \tag{10}$$

Here the overhead dot of above equations represents differentiation with respect to t.

The spatial volume is

$$V = ABC = R^3 \tag{11}$$

Here, R(t) is the scale factor

The constant deceleration parameter q is

$$q = -\frac{R\ddot{R}}{\dot{R}^2} = -(kt + n - 1) \tag{12}$$

The Hubble's parameter H is

$$H = \frac{\dot{R}}{R} = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) \tag{13}$$

where $H_x = \frac{\dot{A}}{A},\ H_y = \frac{\dot{B}}{B}\ H_z = \frac{\dot{C}}{C}$

The scalar expression θ , the shear scalar σ, and mean anisotropy parameter are defined as

$$\theta = 3\left(\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)\right) \tag{14}$$

$$\sigma^2 = \frac{1}{2}\left(\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2\right) - \frac{1}{6}\theta^2 \tag{15}$$

$$A_\alpha = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 \tag{16}$$

where $\Delta H_i = H_i - H$

III. <u>THE FIELD EQUATIONS AND ITS SOLUTION :</u>

The inter dependence relation of q and H is

$$q + 1 = \frac{d}{dt}\left(\frac{1}{H}\right) \tag{17}$$

By the Equation .(2.16), The scale factor R(t) obtained by

$$R(t) = e^{\delta}\left(e^{\int \frac{dt}{\int (1+t)dt + r}}\right) \tag{18}$$

Where $\delta\ and\ r$ are arbitrary constants ,

By Abdussattar, e.t.al,(2011), the proposed choice of q is

$$q + 1 = \frac{-\alpha}{t^2} + \beta \tag{19}$$

where α is a parameter and β is constant,

Depending on different values of α and β , the defined models may vary

Without loss of generality, we consider $\delta = 0\ and\ r = 0$

We obtain the scale factor R(t) as

$$R(t) = \left(t^2 + \frac{\alpha}{\beta}\right)^{1/2\beta} \tag{20}$$

Integrating Eqs.(10) , The relation among A , B and C is

$$A^{m+1} = BC^m \tag{21}$$

It is difficult to calculate the exact values of the unknowns A, B, C, p and ρ from (6) to (9) .

Since , R(t) and the V(t) are proportional to each other and the relation between them is

$$V = R(t)^3 = ABC \tag{22}$$

Clearly V is directly propositional to A, B and C respectively. To find the values of A, B and C , to assume $C = V^K$, where K is constant, in (10) and (11)

The cosmic scale factors for the metric (1) are

$$A = \left(t^2 + \frac{\alpha}{\beta}\right)^{k_1} \tag{23}$$

$$B = \left(t^2 + \frac{\alpha}{\beta}\right)^{k_2} \tag{24}$$

$$C = \left(t^2 + \frac{\alpha}{\beta}\right)^{k_3} \tag{25}$$

Where $k_1 = \frac{3m+3mK-3K}{2\beta(m+2)}$, $k_1 = \frac{3m-3mK-3K-3m^2K}{2\beta(m+2)}$, $k_1 = \frac{3K}{2\beta}$

With the help of (23), (24) and (25) the metric (1) will become

$$\boldsymbol{ds^2 = dt^2 - \left(t^2 + \frac{\alpha}{\beta}\right)^{2k_1} dx^2 - e^{-2x}\left(t^2 + \frac{\alpha}{\beta}\right)^{2k_2} dy^2 - \left(t^2 + \frac{\alpha}{\beta}\right)^{2k_3} e^{-2mx}(t) dz^2} \tag{26}$$

This model represents general class of Bianchi Type dark energy model

III. CHARACTERISTICS AND PHYSICAL BEHAVIOR OF THE MODEL

Equation (26) shows the general class of Bianchi Type dark energy model. The following are the physical and kinematical parameters of the model

$$V = R(t)^3 = ABC = \left(t^2 + \frac{\alpha}{\beta}\right)^{k_4} \tag{27}$$

Where $k_4 = k_1 + k_2 + k_3$

$$H = \frac{1}{3}(H_1 + H_2 + H_3) = k_5\left(\frac{t}{t^2 + \frac{\alpha}{\beta}}\right) \tag{28}$$

where $H_1 = \frac{\dot{A}}{A} = 2k_1\left(\frac{t}{t^2+\frac{\alpha}{\beta}}\right) H_2 = 2k_2\left(\frac{t}{t^2+\frac{\alpha}{\beta}}\right)$, $H_3 = \frac{\dot{C}}{C} = 2k_3\left(\frac{t}{t^2+\frac{\alpha}{\beta}}\right)$ and $k_5 = \frac{2}{3}(k_1 + k_2 + k_3) = \left(\frac{2+Km-Km^2}{\beta(m+2)}\right)$.

$$\theta = 3\left(\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)\right) = 3H = k_6\left(\frac{t}{t^2+\frac{\alpha}{\beta}}\right) \tag{29}$$

Where $k_6 = 3k_5 = \left(\frac{3+3Km-3Km^2}{\beta(m+2)}\right)$

$$\sigma^2 = \frac{1}{2}\left(\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2\right) - \frac{1}{6}\theta^2 = k_7\left(\frac{t^2}{\left(t^2+\frac{\alpha}{\beta}\right)^2}\right) \tag{30}$$

Where $k_7 = 2\left({k_1}^2 + {k_2}^2 + {k_3}^2\right) - \frac{{k_6}^2}{6}$

$$A_\alpha = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 = \left[\frac{4\left({k_1}^2+{k_2}^2+{k_3}^2\right)}{3{k_5}^2} - 1\right] = \text{Constant} \tag{31}$$

$$q = -\frac{R\ddot{R}}{\dot{R}^2} = -(kt + n - 1) \tag{32}$$

The energy density in the model is

$$\rho = -\left[(k_1.k_2 + k_2.k_3 + k_3.k_1)\left(\frac{4t^2}{\left(t^2+\frac{\alpha}{\beta}\right)^2}\right) - \frac{(m^2+m+1)}{\left(t^2+\frac{\alpha}{\beta}\right)^{2k_1}}\right] \tag{33}$$

The pressure in the model is

$$\boldsymbol{p = \left[(a_2 + a_3)\left(\frac{t^2}{\left(t^2+\frac{\alpha}{\beta}\right)^2}\right) + k_2.k_3\left(\frac{4t^2}{\left(t^2+\frac{\alpha}{\beta}\right)^2}\right) - \frac{(m)^2}{\left(t^2+\frac{\alpha}{\beta}\right)^{2k_1}}\right]} \tag{34}$$

Where $a_2 = k_2(k_2 - 1)$, $a_3 = k_3(k_3 - 1)$ and

$k_1 = \frac{3m+3mK-3K}{2\beta(m+2)}$, $k_1 = \frac{3m-3mK-3K-3m^2K}{2\beta(m+2)}$, $k_1 = \frac{3K}{2\beta}$

The EoS parameter is

$$\omega(t) = \frac{p}{\rho} = \frac{-\left[(a_2+a_3)\left(\frac{t^2}{\left(t^2+\frac{\alpha}{\beta}\right)^2}\right)+k_2.k_3\left(\frac{4t^2}{\left(t^2+\frac{\alpha}{\beta}\right)^2}\right)-\frac{(m)^2}{\left(t^2+\frac{\alpha}{\beta}\right)^{2k_1}}\right]}{\left[(k_1.k_2+k_2.k_3+k_3.k_1)\left(\frac{4t^2}{\left(t^2+\frac{\alpha}{\beta}\right)^2}\right)-\frac{(m^2+m+1)}{\left(t^2+\frac{\alpha}{\beta}\right)^{2k_1}}\right]} \quad (35)$$

It may be observed that the initially there is no singularity. And also observed that the volume of the universe increased for large time, it causes accelerated expansion of universe. The parameter $\omega,\ \rho, p\ , H, \theta, and\ \sigma$ are vanish for large values of t and hence they diverge at early stage of the universe.

IV. CONCLUSIONS

We came to observe here that anisotropy dark energy models did a prominent role in discussion of the accelerated expansion of the universe. Here homogeneous and anisotropy General class of Bianchi Type dark energy model with variable EoS parameter have been obtained. It is identified that EoS parameter in this model is function of cosmic time t. therefore the model has no initial singularity. The resultant model obtained is **accelerating, expanding, non-rotating**. Also, we conclude that, the resultant model confirms that high supernova experiment. The behaviors of the model is obtained is similar to the model obtained by the researcher (ref 13).

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Anisotropic Bianchi Type-I Dark Energy Model in Alternative Theory of Gravitation

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Abstract : *A Bianchi Type Dark Energy Model has been derived with scale covariant theory of gravitation formulated by Canuto et al. (Phys. Rev. Lett. 39:429, 1977). And the variable equation of state (EoS) parameter and constant deceleration parameter also investigated The Properties of the model are also discussed.*

***Keywords**: Dark Energy, Bianchi Type models, Scale covariant*

I. Introduction

Currently t0he modified theories of gravitation are playing a prominent role in researching and analyzing the creation of systems and universe evaluation. In general relativity, the scale-covariant theory of gravitation is one of the alternative theories formulated by researchers (Reddy et al. 2012,2014).We have seen the variable gravitational constant G, which is also part of the scale-covariant theory. Within Einstein's field equations, gravitational units are valid and atomic units are valid for physical quantity. In the conformal transformation, these two unit systems are associated with the metric tensors

$$\overline{g_{ij}} = \phi^2\left(x^k\right) g_{ij} \tag{1}$$

Here the Latin indices are 1,2,3,4. The bar and unbar indicates gravitational units, and atomic units respectively .

Accordingly, by the conformal mapping of the type in equation (1), the transformed equations of Einstein are by Canuto et al (1977).

$$R_{ij} - \frac{1}{2} R\, g_{ij} + f_{ij}(\phi) = -8\,\pi\, G(\phi)\, T_{ij} + \Lambda\,(\phi)\, g_{ij} \tag{2}$$

Here,

$$\phi^2 f_{ij} = 2\,\phi\,\phi_{i;j} - 4\,\phi_i\,\phi_j - g_{ij}\left(\phi\phi^{,k}_{;k} - \phi^k \phi_k\right) \tag{3}$$

The semi colon indicates covariant derivative, ϕ_i denotes ordinary derivative with respect to x^i

Here, the energy momentum tensor is T_{ij}, the Ricci tensor R_{ij}, the Ricci scalar is R and the gravitational 'constant' is G and the cosmological constant is Λ .

. The non independent gauge function $\phi\ (0 < \phi < \infty)$ of all space-time coordinates can also be defined as (ref.8)

$$\phi(t) = \left(\frac{k_0}{t}\right)^{\epsilon}\ ,\ \varepsilon = \pm 1,\quad \pm\frac{1}{2} \tag{4}$$

Here, k_0 is constant.

$$\phi \sim t^{\frac{1}{2}} \text{ implies that } \qquad \phi = \phi_0\, t^{\frac{1}{2}} \tag{5}$$

is the best fit to observations.

It is commonly knowledge that, the mysterious dark energy has driven by the universe. rapid expansion . It influences the characterization of dark energy model with the EoS parameter . The EoS parameter $\omega(t) = \frac{p}{\rho}$ is not necessarily constant, where p and ρ are pressure and density of the fluid respectively . Significant introduction as well as excellent review of dark energy models in general relativity is given by several authors (Reddy et al .(2013a,2013b)) in recent years. A dark energy model with axially symmetrical Bianchi type-I has been probed in the scale-covariant theory .S

The paper organizing the following sections : (II) metric and field equations (III) Solution of the field equations. The Characteristic of the model discussed . (V) Conclusions discussed .

II. Metric and Field Equations

Consider an axially symmetric spatially homogeneous Bianchi type-I metric equation is

$$ds^2 = dt^2 - {A_1}^2\, dx^2 - {B_1}^2\,(dy^2 + dz^2) \qquad (6)$$

Here, A_1 and B_1 are functions of cosmic time t

The energy movement tensor is

$$T_j^i = \text{diag}\ [T_0^0, T_1^1, T_2^2, T_3^3\ ] \qquad (7)$$

We can parameterize as follows

$$\left.\begin{aligned} T_j^i &= \text{diag}[\rho, -p_x, -p_y, -p_z\ ] \\ &= \text{diag}[1, -\omega_x, -\omega_y, -\omega_z\ ]\rho \\ &= \text{diag}[1, -\omega, -(\omega+\delta), -(\omega+\gamma)]\rho \end{aligned}\right\} \qquad (8)$$

In equation (8) ρ is the energy density of the fluid with pressures and EoS parameters along cosmic coordinates are p_x, p_y, p_z and ω_x, ω_y, ω_z respectively .

The equation of state (EoS) parameter is

$$\omega = \frac{\rho}{p} \qquad (9)$$

Since we parameterized the deviation from isotropy by choosing $\omega_x = \omega$, and then skewness parameters δ and γ are introduced. Since the equation (6) is axially symmetric spatially homogeneous, $T_2^2 = T_3^3$. Hence, we consider $\delta = \gamma$.

In co-moving coordinate system the field equations (2) and (3) using the equations (7) , (8) and (9) , we have

$$2\frac{\dot{A}_1\dot{B}_1}{A_1B_1} + \left(\frac{\dot{B}_1}{B_1}\right)^2 - \frac{\ddot{\phi}}{\phi} + \frac{\dot{A}_1\dot{\phi}}{A_1\phi} + 2\frac{\dot{B}_1\dot{\phi}}{B_1\phi} + 3\left(\frac{\dot{\phi}}{\phi}\right)^2 = 8\pi G\rho \qquad (10)$$

$$2\frac{\ddot{B}_1}{B_1} + \left(\frac{\dot{B}_1}{B_1}\right)^2 + \frac{\ddot{\phi}}{\phi} - \frac{\dot{A}_1\dot{\phi}}{A_1\phi} + 2\frac{\dot{B}_1\dot{\phi}}{B_1\phi} - \left(\frac{\dot{\phi}}{\phi}\right)^2 = -8\pi G\omega\rho \qquad (11)$$

$$\frac{\ddot{B}_1}{B_1} + \frac{\ddot{A}_1}{A_1} + \frac{\dot{A}_1\dot{B}_1}{A_1B_1} + \frac{\ddot{\phi}}{\phi} + \frac{\dot{A}_1\dot{\phi}}{A_1\phi} - \left(\frac{\dot{\phi}}{\phi}\right)^2 = -8\pi G\rho(\omega+\gamma) \qquad (12)$$

Here the overhead dot indicates derivative with respect to t.

The spatial volume

$$\text{Volume} = A_1{B_1}^2 \qquad (13)$$

The average scale factor

$$R(t) = \left(A_1{B_1}^2\right)^{\frac{1}{3}} \qquad (14)$$

The conventional deceleration parameter q

$$q = -\frac{R\ddot{R}}{(\dot{R})^2} \qquad (15)$$

The scale expansion

$$\theta = \frac{\dot{A}_1}{A_1} + 2\,\frac{\dot{B}_1}{B_1} \qquad (16)$$

The shear scalar

$$\sigma^2 = \frac{1}{3}\left(\frac{\dot{A}_1}{A_1} - \frac{\dot{B}_1}{B_1}\right)^2 \qquad (17)$$

The average anisotropy parameter

$$A_l = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 \qquad (18)$$

where $\Delta H_i = H_i - H \quad (i = 1,2,3)$

The Hubble's parameter

$$H = \frac{\dot{R}}{R} = \frac{1}{3}\left(\frac{\dot{A}_1}{A_1} + 2\frac{\dot{B}_1}{B_1}\right) \qquad (19)$$

$$H = \frac{1}{3}(H_1 + H_2 + H_3) \qquad (20)$$

Here, along the directions of x, y, z. $H_1 = \frac{\dot{A_1}}{A_1}$, $H_2 = H_3 = \frac{\dot{B_1}}{B_1}$, are the directional Hubble's parameters respectively

III. Solution for the model

The equations (10)-(12) are a highly non-linear system of differential equations with 5 unknown equations. In order to obtain explicit solutions to the system, two additional constraints relating to these parameters are required. Clearly , by R Chaubey et al (2017)), the decelerating parameter q is linear in time with a negative slop, and the decelerating parameter q non linear with positive slop and it is a function of time t.

The linear varying deceleration parameter is

$$q = -\frac{R\ddot{R}}{(\dot{a})^2} = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = -\left(\frac{\dot{H}}{H} + 1\right) = -kt + n - 1 \tag{21}$$

In equation (21) k and n are (>) constants.

If q is positive i.e., $n > 1 + kt$, $q > 0$ then the model stands for decelerating universe.

If q is negative i.e., $kt < n < 1 + kt$, $-1 < q < 0$ then the model stands for accelerating universe. From (21) , the average scale factor 'R' is

$$R = (n\,l\,t + c_1)^{\frac{1}{n}}, \quad k = 0,\ n > 0 \tag{22}$$

$$R = c_2\, e^{lt}, \quad k = 0, n = 0 \tag{23}$$

$$R = c_3\, e^{\frac{2}{n}\tanh^{-1}\left(\frac{kt}{n} - 1\right)}, \quad k > 0,\ n > 1 \tag{24}$$

where

$n = (q + 1 + kt) > 0$, c_1, c_2, c_3 are constants

Case1. *If* $k = 0$, $n > 0$ and $A_1 = V^M$, here M is constant

$$A_1 = (nlt + c_1)^{\frac{3M}{n}}, \quad B_1 = (nlt + c_1)^{\frac{3M(1-M)}{2n}} \tag{25}$$

Hence the metric (6) is

$$ds^2 = dt^2 - (nt)^{\frac{3M}{n}} dx^2 - (nt)^{\frac{3M(1-M)}{2n}}(dy^2 + dz^2) \tag{26}$$

with out loss of generality take l=1 and c_1 =0

Characteristics of the model for case 1 .

The metric equation (26) represents an axially symmetric Bianchi type-I dark energy cosmological model when $k = 0$, $n > 0$.

The energy density

$$\rho = \frac{1}{8\pi G}\left[\left(\frac{9M(1-M}{n^2}\right) + \left(\frac{3M(1-M)}{2n}\right)^2 - \left({\phi_0}^2\epsilon^2 - \phi_0\epsilon\right) + \left(\frac{3\phi_0\epsilon}{n}\right) + \left(\frac{3\phi_0\epsilon M(1-M)}{n}\right) + 3\left({\phi_0}^2\epsilon^2\right)\right]\left(\frac{1}{t^2}\right) \tag{27}$$

The EoS parameter is

$$\omega = -\frac{1}{8\pi G\rho}\left[3M(1-M)\left(\frac{3M(1-M)-1}{2n^2}\right) + \left({\phi_0}^2\epsilon^2 - \phi_0\epsilon\right) - \left(\frac{3\phi_0\epsilon}{n}\right) + \left(\frac{3\phi_0\epsilon M(1-M)}{n}\right) - \left({\phi_0}^2\epsilon^2\right)\right]\left(\frac{1}{t^2}\right) \tag{28}$$

The Skewness parameter is

$$\gamma = \delta = \frac{-1}{8\pi G\rho}\left[6M(1-M)\left(\frac{3M(1-M)-n}{2n^2}\right) + 3M\left(\frac{3M-n}{n^2}\right) + \left(\frac{3M(1-M)}{2n}\right)^2 + 2\left(\frac{3\phi_0\epsilon M(1-M)}{n}\right)\right]\left(\frac{1}{t^2}\right) \tag{29}$$

$$\phi = \phi_0 t^{\epsilon}, \quad \epsilon = \pm 1, \pm\frac{1}{2} \tag{30}$$

$$V = (n\,t)^{\frac{3}{n}} \tag{31}$$

$$\theta = \left(\frac{3(2M - M^2)}{n}\right)\frac{1}{t} \tag{32}$$

$$\sigma^2 = \left(\frac{9(M + M^2)^2}{n^2}\right)\left(\frac{1}{t^2}\right) \tag{33}$$

$$A_l = \frac{-6M^4 + 18M^3 - 15M^2}{2M^2(2-M)^2} \tag{34}$$

The Hubble's parameter

$$H = \left(\frac{(2M - M^2)}{n}\right)\left(\frac{1}{t}\right) \quad (35)$$

Where $H_x = \frac{3M}{nt}$ and $H_y = H_z = \frac{3M(1-M)}{2(nt)}$ (36)

Also

$$\frac{\sigma^2}{\theta^2} = \frac{(1+M)^2}{(2-M)^2} = \text{constant} \quad (37)$$

***Case2*. If $k = 0$, $n = 0$ and $A_1 = V^M$, where M is constant**

Since $R = c_2 e^{lt} = V^{\frac{1}{3}}$ (38)

$$A_1 = \left(c_2 e^{lt}\right)^{3M}, B_1 = \left(c_2 e^{lt}\right)^{\frac{3(M-1)}{2}} \quad (39)$$

Hence the metric (6) is

$$ds^2 = dt^2 - (e^t)^{3M} dx^2 - (e^t)^{\frac{3(M-1)}{2}}(dy^2 + dz^2) \quad (40)$$

with out loss of generality take l=1 c_2 =1

<u>Characteristics of the model for case-2</u>

The metric equation (40) represents an axially symmetric Bianchi type-I dark energy cosmological model if $k = 0$, $n = 0$.

$$\rho = \frac{1}{8\pi G}\left[\left(\frac{9M(1-M)}{e^{2t}}\right) + \left(\frac{3(M-1)}{e^t}\right)^2 - \frac{({\phi_0}^2\epsilon^2 - \phi_0\epsilon)}{t^2} + \left(\frac{3\phi_0\epsilon M}{te^t}\right) + \left(\frac{3\phi_0\epsilon M(M-1)}{te^t}\right) + \frac{3({\phi_0}^2\epsilon^2)}{t^2}\right] \quad (41)$$

$$\omega = \frac{-1}{8\pi G\rho}\left[\left[\frac{1}{2}\left(\frac{3(M-1)}{e^t}\right)^2 - \left(\frac{3(M-1)}{e^t}\right)\right] + \left(\frac{3(M-1)}{e^t}\right)^2 + \frac{({\phi_0}^2\epsilon^2 - \phi_0\epsilon)}{t^2} - \left(\frac{3\phi_0\epsilon M}{te^t}\right) + \left(\frac{3\phi_0\epsilon M(M-1)}{te^t}\right) - \frac{({\phi_0}^2\epsilon^2)}{t^2}\right] \quad (42)$$

$$\gamma = \delta = \frac{1}{8\pi G\rho}\left[\left[\frac{1}{2}\left(\frac{3(M-1)}{e^t}\right)^2 - \left(\frac{3(M-1)}{e^t}\right)\right] + \left(\frac{3(M-1)}{e^t}\right)^2 + \left(\frac{3\phi_0\epsilon M(M-1)}{te^t}\right) - \left(\frac{3M}{e^t}\right)^2 - \left(\frac{9M(1-M)}{e^{2t}}\right) - \left(\frac{3\phi_0\epsilon M}{te^t}\right)\right] \quad (43)$$

$$\phi = \phi_0 t^\epsilon, \quad \epsilon = \pm 1, \pm\frac{1}{2} \quad (44)$$

$$V = (e^t)^3 \quad (45)$$

$$a = e^t \quad (46)$$

$$H = \left(\frac{2M-1}{e^t}\right) \quad (47)$$

$$\theta = \left(\frac{2M-1}{e^t}\right) \quad (48)$$

$$\sigma^2 = \frac{3}{4}\left(\frac{M+1}{e^t}\right)^2 \quad (49)$$

$$A_l = \frac{-3M^2 + 10M - 3}{(2M-1)^2} \quad (50)$$

Also

$$\frac{\sigma^2}{\theta^2} = \frac{3}{4}\frac{(1+M)^2}{(2M-1)^2} = \text{constant} \quad (51)$$

***Case3*. If $k > 0$, $n > 1$, and $A_1 = V^M$, where M is constant**

$$\text{since}, R = c_3 e^{\frac{2}{n}\tanh^{-1}\left(\frac{kt}{n}-1\right)} = V^{1/3}$$

$$A_1 = \left(c_3 e^{\frac{2}{n}\tanh^{-1}\left(\frac{kt}{n}-1\right)}\right)^{3M},$$

$$B_1 = \left(c_3 e^{\frac{2}{n}\tanh^{-1}\left(\frac{kt}{n}-1\right)}\right)^{\frac{3(M-1)}{2}} \quad (52)$$

Hence the metric (6) is

$$ds^2 = dt^2 - \left(e^{\frac{2}{n}\tanh^{-1}\left(\frac{kt}{n}-1\right)}\right)^{3M} dx^2 - \left(e^{\frac{2}{n}\tanh^{-1}\left(\frac{kt}{n}-1\right)}\right)^{\frac{3(M-1)}{2}} (dy^2 + dz^2) \quad (53)$$

with out loss of generality take c_3 =1

Characteristics of the model for case 3.

The equation (53) shows an axially symmetric Bianchi type-I with dark energy cosmological model if $k > 0$, $n > 1$.

$$\rho = \frac{1}{8\pi G}\left[\left(\frac{9M(M-1)n^4k^2}{2}\right)\left(\frac{1}{(k^2t-2kn)\tanh^{-1}\left(\frac{kt}{n}-1\right).}\right)^2 + \left(\frac{3(M-1)n^2k}{2(k^2t-2kn)\tanh^{-1}\left(\frac{kt}{n}-1\right).}\right)^2 - \left({\phi_0}^2\epsilon^2-\phi_0\epsilon\right) + \left(\frac{3Mn^2k\phi_0\epsilon}{2(k^2t-2kn)\tanh^{-1}\left(\frac{kt}{n}-1\right).}\right) + \left(\frac{3(M-1)n^2k\phi_0\epsilon}{(k^2t-2kn)\tanh^{-1}\left(\frac{kt}{n}-1\right).}\right) + 3\left({\phi_0}^2\epsilon^2\right)\right]\left(\frac{1}{t^2}\right) \tag{54}$$

$$\boldsymbol{\omega} = -\frac{1}{8\pi G\rho}\left[\frac{3(M-1)k\left[n^2(2k^2t-2kn)\tanh^{-1}\left(\frac{kt}{n}-1\right)+2(k^2t^2-2knt)^2\right]}{\left((k^2t^2-2knt)\tanh^{-1}\left(\frac{kt}{n}-1\right)\right)^2.} + 3\left(\frac{3(M-1)n^2k}{2(k^2t^2-2knt)\tanh^{-1}\left(\frac{kt}{n}-1\right).}\right)^2 + \left(\frac{{\phi_0}^2\epsilon^2-\phi_0\epsilon}{t^2}\right) - \left(\frac{3Mn^2k\phi_0\epsilon}{2t(k^2t^2-2knt)\tanh^{-1}\left(\frac{kt}{n}-1\right).}\right) + 2\left(\frac{3(M-1)n^2k\phi_0\epsilon}{2t(k^2t^2-2knt)\tanh^{-1}\left(\frac{kt}{n}-1\right).}\right) - \left(\frac{{\phi_0}^2\epsilon^2}{t^2}\right)\right] \tag{55}$$

$$\gamma = \delta = \frac{-1}{8\pi G\rho}\left[\frac{3(M-1)k\left[n^2(2k^2t-2kn)\tanh^{-1}\left(\frac{kt}{n}-1\right)+2(k^2t^2-2knt)^2\right]}{\left((k^2t^2-2knt)\tanh^{-1}\left(\frac{kt}{n}-1\right)\right)^2.} + 2\left(\frac{3(M-1)n^2k}{2(k^2t^2-2knt)\tanh^{-1}\left(\frac{kt}{n}-1\right).}\right)^2 + 2\left(\frac{3(M-1)n^2k\phi_0\epsilon}{2t(k^2t^2-2knt)\tanh^{-1}\left(\frac{kt}{n}-1\right).}\right) - \frac{3Mk\left[n^2(2k^2t-2kn)\tanh^{-1}\left(\frac{kt}{n}-1\right)+2(k^2t^2-2knt)^2\right]}{\left((k^2t^2-2knt)\tanh^{-1}\left(\frac{kt}{n}-1\right)\right)^2.} - \left(\frac{3Mn^2k\phi_0\epsilon}{2t(k^2t^2-2knt)\tanh^{-1}\left(\frac{kt}{n}-1\right).}\right)\right] \tag{56}$$

$$\phi = \phi_0 t^\epsilon, \quad \epsilon = \pm 1, \pm\frac{1}{2} \tag{57}$$

$$V = \left(c_3 e^{\frac{2}{n}\tanh^{-1}\left(\frac{kt}{n}-1\right)}\right)^{(6M-1)} \tag{58}$$

$$\theta = \left(\frac{3(3M-1)n^2k}{2(k^2t^2-2knt)\tanh^{-1}\left(\frac{kt}{n}-1\right).}\right) \tag{59}$$

$$\sigma^2 = 3\left(\frac{n^2k}{2(k^2t^2-2knt)\tanh^{-1}\left(\frac{kt}{n}-1\right).}\right)^2 \tag{60}$$

$$A_l = \frac{5-12M}{(3M-1)^2} \tag{61}$$

$$H = \left(\frac{(3M-1)n^2k}{2(k^2t^2-2knt)\tanh^{-1}\left(\frac{kt}{n}-1\right).}\right) \tag{62}$$

Also

$$\frac{\sigma^2}{\theta^2} = \frac{1}{3(3M-1)^2} = \text{constant} \tag{63}$$

In all three cases ,Clearly It can be observed that at early stage of universe the models (26, 40 , 53) free from initial singularity. The physical quantities ρ, ω, γ , θ , σ^2, H are diverges and they become vanish for infinite value of t. The scalar field function ϕ tends to infinity when $\epsilon = 1, \frac{1}{2}$ while it is zero when $\epsilon = -1, -\frac{1}{2}$. As t increases the spatial volume is also increases, which represents the universe is expanding. Since, A_l is constant, the model is uniform entirely the evolution of the universe. Since, $\frac{\sigma^2}{\theta^2} = \text{constant}$, the model is anisotropy model at large values of t.

IV. Conclusions

Dark energy cosmological models plays an prominent place in the general relativity which is observed that, the universe. is accelerated. The models developed with alternative theories of gravitation are obtaining more importance.. It is also observed that, the required model is anisotropic and free from initial singularity. All the physical parameters of this model are diverse at the initial epoch, $t = 0$ and tend to zero for large values of t.

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